# XI. *On the High-Temperature Standards of the National Physical Laboratory: an Account of a Comparison of Platinum Thermometers and Thermojunctions with the Gas Thermometer.*

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## CONTENTS.

## I. *Introduction.*

IN a paper "On the Comparison of Gas and Platinum Thermometers," read before the Royal Society in 1900,[*] Dr. P. CHAPPUIS and the author described a series of experiments in which several platinum-resistance thermometers, constructed of wire of specially high purity, were compared with the gas thermometer at a number of steady temperatures from below zero to above the boiling-point of sulphur, and in one set of measurements to just short of 600° C.

The results were such as to substantially confirm the conclusion of CALLENDAR and GRIFFITHS that the indications of platinum thermometers may be reduced to the normal scale by the employment of CALLENDAR's well-known difference formula

$$d \equiv T - pt = \delta \left[ \left( \frac{T}{100} \right)^2 - \frac{T}{100} \right]$$

where $d =$ the difference between T, the temperature on the normal scale, and $pt =$ the "platinum" temperature. The constant $\delta$ for pure platinum wires is approximately 1·5, the three temperatures chosen for its determination being 0°, 100° and the boiling-point of sulphur.

The paper concludes with the sentence, "until further investigations have been made as to the relations of the various gas scales at high temperatures and as to the influence of the initial pressure and the effect of impurities and traces of water vapour in the gases employed, and until exact determinations have been made up to high temperatures of the coefficient of expansion of the material used as thermometric reservoir, we think that for the purposes of high-range thermometry a scale deduced by the parabolic formula from that of the platinum thermometer will suffice. In the present state of our knowledge any attempt to improve on such a thermometric scale would be attended with such uncertainties as would probably render it futile."

Since that time, however, a substantial advance has been made in our knowledge, direct determinations of the expansion of porcelain up to high temperatures having been made by different observers, namely, Mr. BEDFORD,[†] at Cambridge, and Messrs. HOLBORN and DAY at the Reichsanstalt.[‡] A discussion by Dr. CHAPPUIS of the results obtained by these observers and their influence on high-range thermometry is found in the 'Philosophical Magazine,' (5), October, 1900, and February, 1902.

An examination of the difference formula for the platinum thermometer shows that it can only represent a physical reality over a limited range, the value of $pt$ for a wire having a $\delta$ of 1·5 reaching a maximum about 1700° $pt$, a value numerically not far exceeding such as may safely be attained. It would not therefore be surprising if the formula which actually holds remarkably closely at low ranges should be found to

[*] 'Phil. Trans.,' A, vol. 194, pp. 37–134.
[†] BEDFORD, 'Proc. Phys. Soc.,' XVII., Part III., p. 148, and 'Phil. Mag.'
[‡] HOLBORN and DAY, 'Ann. Phys.,' vol. 6, 1901, p. 136.

give erroneous results at temperatures well below the maximum to which the materials used in the construction of a platinum thermometer can be subjected without injury. The investigations dealt with in the present paper have been carried out at the National Physical Laboratory during the past two years, and consist mainly of a continuation of the work of CHAPPUIS and the author on the platinum thermometer, testing up to 1000° C. the validity of the difference formula for two thermometers made of representative platinum wires of high purity, by comparison of these instruments with the constant volume gas thermometer. With these instruments were also compared simultaneously standard thermojunctions, whose electromotive force at a series of temperatures had been determined with special care at the Reichsanstalt at Charlottenburg.

## II. *The Gas Thermometer.*

The gas thermometer employed for this work is a duplicate of the one used by HOLBORN and DAY at the Reichsanstalt. It was obtained from the same maker, FUESS, of Berlin, and was presented to the laboratory by Sir ANDREW NOBLE. For this munificent gift and for the kindly assistance and advice rendered by the President of the Reichsanstalt, Dr. KOHLRAUSCH, and by Dr. HOLBORN in procuring for us the gas thermometer, thermocouple, wire and materials for the construction of electric furnaces, the laboratory is greatly indebted.

The instrument is specially designed for rapid work at high temperatures, and was arranged so that measurements could be made with any desired initial pressure and with bulbs of different materials. The principle employed by CHAPPUIS, in the two gas thermometers at Sèvres, of making all the measurements depend upon the determination of a single length, though undoubtedly capable of giving by far the most accurate results, becomes somewhat inconvenient when great changes of pressure are needed. For this reason, therefore, in the present apparatus the manometer is arranged so as to measure directly the difference of height between the level of a very short metal point, to which the mercury in the closed limb A, fig. 1, is adjusted, and the mercury surface in the long tube B, which during the measurements communicates with the atmosphere by the tap H.*

The tubes A and B communicate by means of cone joints with the lower part of a closed iron reservoir in the base plate of the apparatus, into which mercury can enter from the upper reservoir G by means of the long tube C and steel tap D. The fine adjustment of the height of the mercury to the point in A is made by a steel screw with capstan-shaped head projecting from the bottom of the apparatus and working on a thin steel diaphragm let into the bottom of the reservoir.

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* In the original form of the apparatus, tube A was joined to the reservoir below by a large three-way glass tap, through the side tube of which the filling of the gas into the reservoir was made. It was found, however, that this tap was a source of danger in the measurements, the results of one set of comparisons
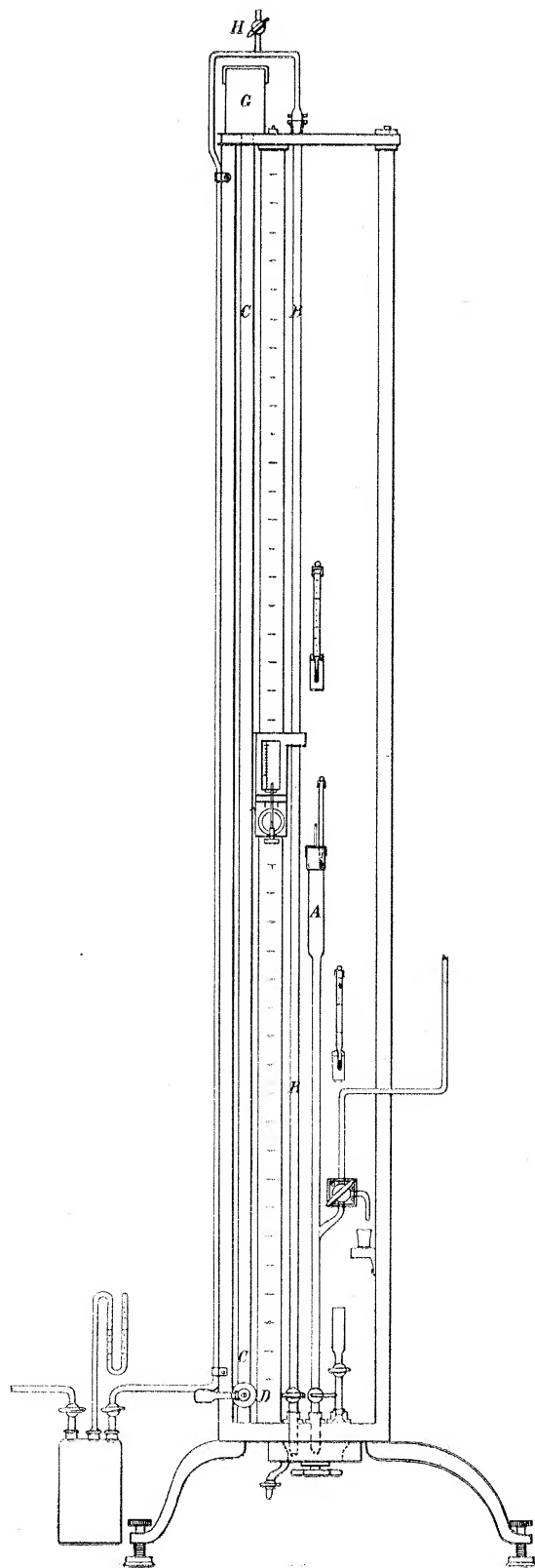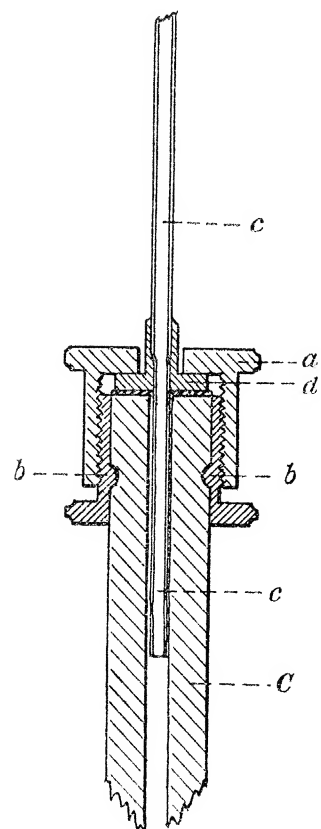
Fig. 1.



Fig. 2.

Joint between platinum capillary
and neck of reservoir.

*a*, metal cap; *b*, split metal collar;
C, porcelain capillary; *c*, plati-
num capillary, in this case drawn
down to smaller size for lower
4 or 5 centims.; *d*, metal washer
soldered to platinum capillary.

### III. *The Scale, &c.*

The manometer scale, nearly two metres long, is graduated into millimetres and carries a slider with vernier and fine adjustment screw by which the position of the mercury column in B could be read directly to ·02 millim. Over the part of the scale used in these experiments no corrections were required.

Accurate setting of the mercury to the point in the closed limb of the manometer was facilitated by the use of a small short-focus telescope clamped to the stand. Measurements of the height of this point on the scale, as given by the vernier readings when the sighting edge of the slider was brought in line with the point, were taken frequently throughout the work. The transfer of level was effected by a Quincke microscope fixed on a plate-glass table which could be attached to the gas thermometer supports. This constant was different in the several sets owing to the tube A having for different reasons been several times rebuilt.

### *The Reservoirs and their Attachment, &c.*

For some early experiments up to 200° C. reservoirs of normal Jena glass 16‴ were fixed having a long capillary stem attached by fusion. For the later series at high temperatures reservoirs of Berlin porcelain, similar to those formerly employed by CHAPPUIS and HARKER, were kindly ordered for us by Dr. HOLBORN at the Berlin factory. They were made in manner described by HOLBORN and DAY in their paper in 'Wied. Ann.,' vol. 68, pp. 830, 831.

For attachment of the reservoir to the platinum capillary leading to the manometer joints made by sealing-wax or cement are too treacherous to be trusted for long periods, so the joints were in all cases made in the manner shown in fig. 2. The brass washer soldered on to the platinum capillary is clamped down tight against the ground end of the porcelain or glass neck of the reservoir by a hollow cap which screws on to the two halves of a split brass collar, a thin leather washer, on which is smeared a little marine glue, being interposed to make the joint. The collar is prevented from sliding upon the neck by fitting closely into a rounded channel cut into the glass or porcelain by a small emery wheel. This form of joint has given perfect satisfaction.

being lost through the plug being forced outward by the mercury pressure. Although the possibility of the recurrence of this accident was avoided by the subsequent addition of a spring frame arrangement embracing the plug of the tap, it was found extremely difficult to arrange the lubrication to be free enough to permit of its constant use for shutting off communication between the thermometer and the bulb and at the same time to prevent very slow inward leakage of air into A through the tap when the bulb was under low pressure. The apparatus was therefore modified by putting into tube A a plain steel tap and introducing a smaller and perfectly ground three-way tap into the side tube, where it was not needed after the bulb was once filled. Under these conditions the whole arrangement could be kept perfectly gas tight.

IV. *Barometer and Auxiliary Measurements.*

The barometer used was a Fortin with $\frac{3}{4}$-inch tube, made by HICKS and standardised at Kew. It was compared from time to time with an old Royal Society's standard and found to have the same correction, — ·05 millim.

The temperatures of the mercury columns in the gas thermometer were obtained by means of two auxiliary thermometers disposed at convenient heights, having their bulbs immersed in short test tubes filled with mercury. These are called (2) and (3) in the examples of gas thermometer readings given later. The temperature of that part of the dead space over the mercury in tube A is given by a thermometer inserted into the metal stopper carrying the reference point. This thermometer is labelled (1). The temperature of the platinum capillary is kept constant by surrounding it with a water-jacket of rubber tube, through which a circulation of water from a supply vessel kept in the room is maintained during the experiments, the temperature of the flow being given by the thermometer (4).

In making the correction of the pressures of the various mercury columns to latitude 45° and sea level, the latitude of the thermometric laboratory was taken as 51° 26′ 47″, and the height above sea level of the cistern of the barometer as 10 metres.

The relation

$$\frac{\text{G latitude Bushy House}}{\text{G latitude 45}°} \text{ was taken as } 1\cdot000561.$$

V. *Preparation of the Gases.*

The nitrogen used in the earliest experiments was obtained from atmospheric air by the ordinary method of absorbing the carbonic acid by potash and the oxygen by hot copper, but for the sake of continuity with the work of CHAPPUIS and the author, in which chemical nitrogen was used in all the high-temperature experiments, the method of preparation employed by them was used in all the later work. Into a flask containing 100 grammes of potassium bichromate, dissolved in air-free distilled water, was introduced gradually by means of a tap-funnel a strong solution containing 100 grammes nitrate of soda and 100 grammes nitrate of ammonia. When gently heated on a water-bath this mixture gives off a steady stream of nitrogen, which is passed over hot copper and copper oxide to destroy any oxides of nitrogen it may contain, and collected over weak boiled potash solution in a gas holder. From this it passes through solutions of silver nitrate and strong potash, boiled sulphuric acid, and over solid caustic potash to a larger U-tube containing phosphoric anhydride, where it is left several days before use, or is collected in larger quantity in the dry gas holder over mercury. The tightness of all the joints and taps was ultimately made so perfect that no measurable inward leak of air could be detected by the attached manometer, even when the whole was left standing under reduced pressure for some months. A sketch of the gas preparation apparatus is shown in fig. 3.
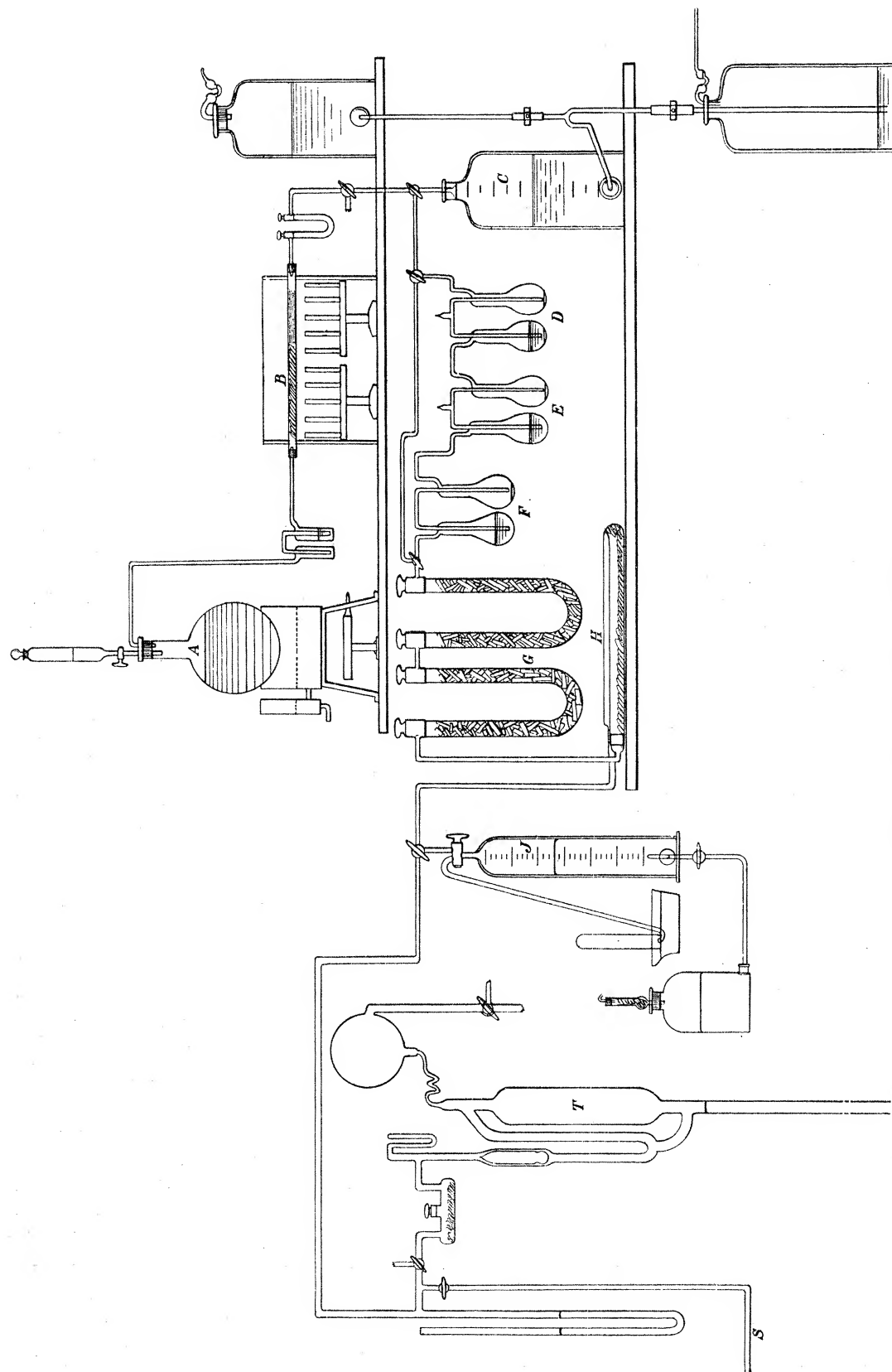
Fig. 3. Gas preparation apparatus.

A, generating flask ; B, tube containing copper or copper oxide ; C, wet gasholder ; D, silver nitrate solution ; E, strong caustic potash ; F, strong sulphuric acid ; G, solid potash and anhydrous baryta ; H, phosphoric anhydride ; J, mercury gasholder ; T, Töpler pump ; S, tube to gas thermometer.

## VI. *Filling of the Reservoir.*

The experiments of CHAPPUIS and the author, 'Phil. Trans.,' vol. 194, p. 93, showed how important it is that previous to and during the filling of the reservoir with gas the whole should be heated to as high a temperature as possible—indeed that with a reservoir of verre dur it was impossible to obtain an accurate value for a temperature in the neighbourhood of the boiling-point of sulphur unless it had been previously heated for a considerable period to a temperature considerably higher than this. Although very little is known as to the behaviour with change of temperature and pressure of the films of condensed gas, which by their surface tension adhere closely to the walls of a glass or porcelain vessel, and how they influence the coefficient of expansion of any gas enclosed in it, it seems at any rate advisable to remove the existing film in any reservoir to be used for high-temperature work as far as possible by repeated heating and evacuation. Accordingly for each new filling during this work the procedure was to heat the reservoir very strongly, exhausting it and the connecting tubes leading to the pump so thoroughly that an electrodeless vacuum-tube attached by a side tube showed strong green fluorescence both when the reservoir was hot and the next day after cooling. This was only attained after much labour by taking special care in the manipulation of all the taps and joints in the circuit, and by blowing together into one piece the whole of the long canal leading from the gas thermometer in the main laboratory to the pump and gas preparation apparatus in a small room on an upper floor. After the apparatus had stood this test the gas was filled into the reservoir to 100 to 200 millims. pressure at least three times in each case before the final filling and pressure adjustment were made, the bulb being heated meanwhile as strongly as expedient in the particular case.

## VII. *Dilatation of the Porcelain Reservoir.*

In 1898 and 1899, CHAPPUIS and HARKER, in the absence of precise data as to the expansion at high temperatures of the reservoirs of Berlin porcelain they employed in their gas thermometer, were obliged to content themselves with extrapolation of the expression for the expansion obtained by them from determinations made in the Benoit-Fizeau dilatometer between the limits 0° and 100°, confirmed by further experiments made by a weight thermometer method. The values for the expression they employed seemed at the time to be amply confirmed by the measurements of HOLBORN and WIEN, made in 1892, which gave for the mean coefficient of linear expansion between the limits 0° and 600° the value $4400 \times 10^{-9}$, the value calculated by CHAPPUIS and HARKER for the same temperature limits being $4484 \times 10^{-9}$. On the other hand, the later experiments of HOLBORN and DAY made on Berlin porcelain rods heated horizontally in an electric furnace show fairly conclusively that the expansion of porcelain cannot be represented by any simple function for more than a

few hundred degrees, and that therefore an expression deduced empirically cannot be applied outside the limits of actual experiment without risk of serious error.

To obtain the most probable value over the whole range for the expansion of Berlin porcelain, a combination was made of the values obtained directly by two independent methods over the range 0° to 100° C. by CHAPPUIS and HARKER with those obtained for higher temperatures by HOLBORN and DAY. As pointed out by CHAPPUIS in the paper in the ' Philosophical Magazine' alluded to in the introduction, the formula of HOLBORN and DAY undoubtedly gives too high values below 250° C. This has just been confirmed by the experiments of SCHEEL, published in vol. 4 of the ' Wiss. Abh. der Reichsanstalt,' whose results over the range 14° to 100° obtained in the Fizeau dilatometer are in close agreement with those of CHAPPUIS and HARKER.

I have therefore taken for this work the following values of the mean dilatation between 0° and T°, and from them calculated a table giving the volume at T° of a reservoir, whose volume at 0° is unity, for temperatures up to 1100° C.

MEAN Dilatation of Berlin Porcelain between 0° and T°.

| T. | Dilatation. $\times 10^{-8}$. | T. | Dilatation. $\times 10^{-8}$. |
|---|---|---|---|
| 0 | 269 | 600 | 363 |
| 100 | 299 | 700 | 374 |
| 200 | 318 | 800 | 385 |
| 300 | 329 | 900 | 397 |
| 400 | 340 | 1000 | 408 |
| 500 | 352 | 1100 | 419 |

whence volume at T° of reservoir whose volume at 0° = 1

| | |
|---|---|
| 0 | 1·000000 |
| 100 | 1·000897 |
| . . . | . . . . . |
| . . . | . . . . . |
| 1000 | 1·013834, &c. |

## VIII. *Pressure Coefficient of the Reservoirs.*

In the work of CHAPPUIS and HARKER account was taken of the change of volume of the bulb produced by changes of internal pressure. The value of the change of volume was deduced from observations made by varying the external pressure on the reservoir at ordinary temperatures. With the thick-walled reservoirs here employed the whole effect is only very small, a change of 1 metre in the internal pressure producing a change of about $\frac{1}{40,000}$ of the whole volume of the reservoir at ordinary temperatures. The effect of high temperatures in changing the elastic constants of the porcelain here involved being quite unknown, it was deemed justifiable to omit this correction altogether for the present high-temperature experiments.

## IX. *The Platinum Thermometers.*

The platinum thermometers chosen for the comparison at high temperatures with the gas thermometer were one of a group of six constructed in 1902 at the laboratory for the British Association and described as $BA_2$ in the report of the Electrical Standards Committee for 1903, and the much older one, lettered $K_2$, which had been employed by CHAPPUIS and HARKER in their work at Sèvres. Since the date of the Sèvres comparisons in 1898 and 1899 this thermometer had been preserved as a standard and used comparatively seldom.

Its characteristic dimensions are :—

> Length from end of porcelain tube to below the wooden collar, 35·5 centims.
> External diameter of tube, 11·5 millims.

The head was of the old pattern with terminals, the leads being entirely of platinum, ·020 inch diameter, and the "bulb" wire, ·006 inch diameter.

Thermometer $BA_2$ is similar in design to CHAPPUIS' and HARKER'S thermometers $K_8$ and $K_9$, illustrated in their paper, the head being arranged to be capable of standing a vacuum for a considerable time.

Its dimensions were as follows :—

> Length of tube to under head = 43·5 centims.
> Internal diameter of porcelain tube = 11·5 millims.
> Length of mica cross = 40 millims.
> Thickness of "bulb" wire = ·006 inch.
>      „      "lead"    „    = ·020    „

Both "bulb" and leads were constructed from the stock of wire of extra high purity prepared by JOHNSON and MATTHEY for the British Association. Details as to the construction of these British Association thermometers are given in the appendix to the Report of the British Association, 1903.

## X. *The Resistance Box for Platinum Thermometry.*

The resistance box used was the one originally described by GRIFFITHS in 'Nature,' November 14, 1895, pp. 39 to 46, which had subsequently been modified by the Cambridge Scientific Instrument Company by the substitution of Doulton-ware plug holders for the usual brass blocks to which the individual resistances were connected. In this form the box was used by CHREE in his investigations described in 'Proc. Roy. Soc.,' vol. 67, pp. 3–58.

During the period covered by the present work the coils were standardised in the manner described by HARKER and CHAPPUIS (*loc. cit.*, p. 52), and great care was taken throughout as to the cleanliness of the plugs and to prevent the accumulation of the black deposit which has a tendency to form in the plug holes. The box unit is very exactly ·01 ohm, and the nine coil values are arranged on the binary scale

from 5 up to 1280 units, with an additional coil of 100 units for the determination of the fundamental interval. The coils are of fine platinum-silver wire wound into loose hanks not paraffined, so as to follow with as little lag as possible the temperature of the massive copper tank in which they are placed. The working of the resistance box has under the conditions which obtained for this work been sufficiently good to render unlikely the introduction of any systematic errors. The sensitivity of the platinum thermometer bridge was such that even at the highest ranges a *change* of about ·01° C. could be detected, though an accuracy of ·1° at 1000° C. is probably about the limit in the *absolute* measurement of temperature even under favourable conditions with resistance coils of platinum-silver, such as those employed here, with a temperature coefficient of ·00026 per degree C.

The flexible leads which were never detached from the thermometers during any series were carefully checked before and after the experiments, and the possible checks, got by using different combinations of coils in measurement of the same temperature, always agreed reasonably well.

## XI. *The Potentiometer for Thermocouple Measurements.*

The potentiometer used in these experiments was specially designed and made in the laboratory for accurate thermocouple work, and was described in detail in a paper in the ' Phil. Mag.' for July, 1903. It is direct reading, and is arranged so that for
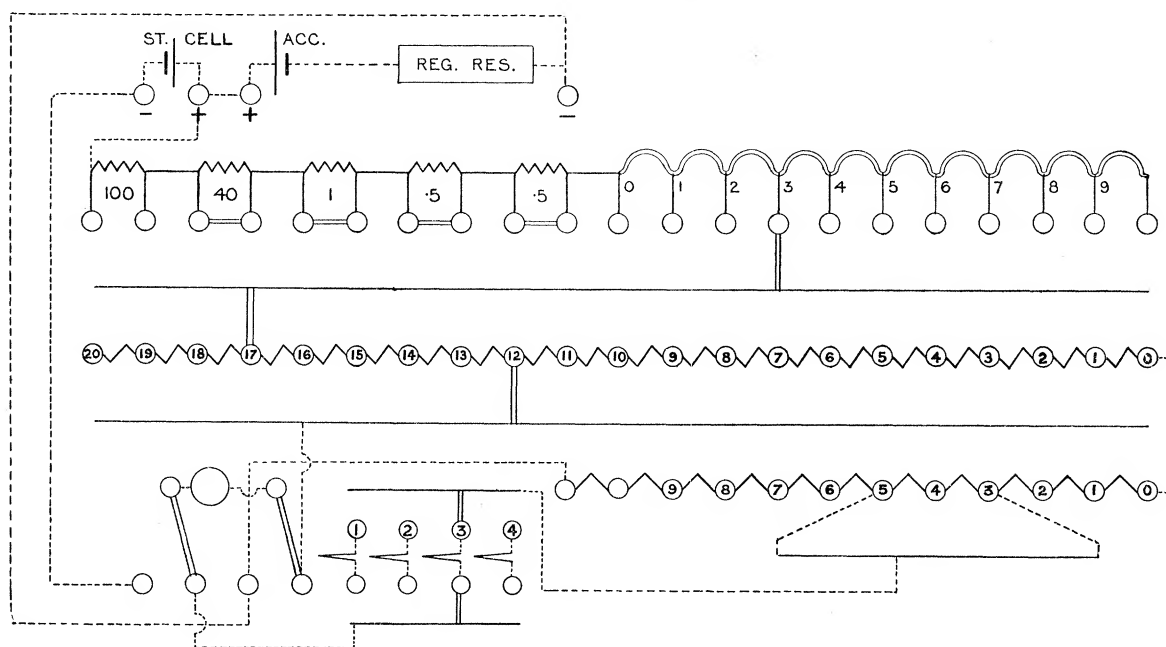


Fig. 4. Connections to potentiometer.

ordinary thermoelectric work where the maximum electromotive force to be measured is of the order of ·02 volt the individual settings could be made to about ·1 microvolt if required. A diagrammatic representation of the connexions is shown in fig. 4 and

a plan of the top of the instrument in fig. 5.   It is designed with a view of rendering possible the use of a short slide-wire of large cross section.   The balancing coils, on which the fall of potential is adjusted to a definite value, are in two rows, the centre
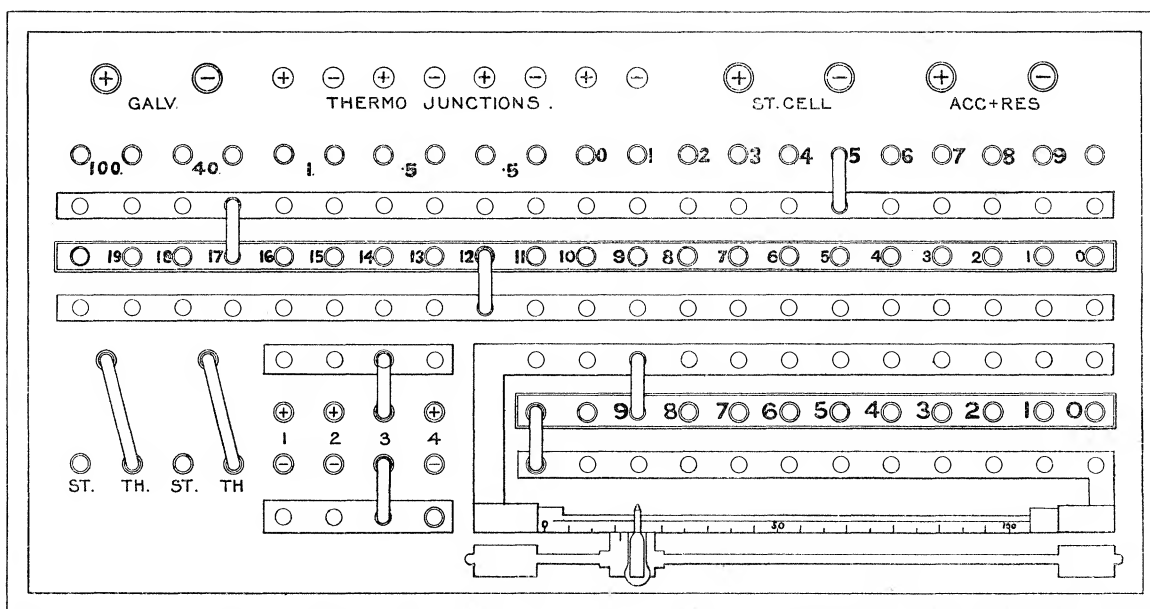


Fig. 5.   Plan of potentiometer.

row of the box consisting of 20 coils of $\frac{1}{10}$ of an ohm each.   In series with these is a second row, immediately behind the bridge-wire, consisting of 11 coils of $\frac{1}{100}$ of an ohm each.   By means of an arrangement of thick copper bars connected with the ends of the slide-wire, which has a total resistance of ·02 ohm, any two adjacent coils of this latter series may be put in parallel with the slide-wire. The 11 coils of ·01, two of which are thus shunted, are therefore always exactly equivalent to ·1 ohm.

For ordinary thermoelectric work the fall of potential along these two sets of coils is adjusted so that each of the back row represents 1000 microvolts, each coil of ·01 ohm being therefore 100 microvolts.   The slide-wire, 200 millims. long, is provided with a divided scale on which $\frac{1}{1000}$ part of its length can be easily estimated.   It will be seen that the slide-wire thus connected acts like a vernier to the small coils.

The adjustment of the electromotive force is made by a standard Clark or Weston cell and the auxiliary set of coils in the back row, a feature of the instrument being that without any external alteration either form of standard may be used at will. The five coils to the left are permanently connected in series, but are arranged so that any coil may be cut out of circuit when required.   Their values are 100, 40, 1, ·5, and ·5 ohms respectively.   Those to the right are a set of 10 simple series coils of ·01 ohm each, arranged so that a connexion can be taken from any one of them to

the long copper bar just in front. Suppose we wish to use as a standard a Clark cell whose electromotive force at the prevailing temperature is 1·4333 volts. It is obvious that we may make the fall of potential over the ·1 ohm balancing coils have the desired value of 1000 microvolts by putting into circuit coils 100, 40, 1, ·5, and three of the ·01 series, leading from the third hundredth by means of the copper bar to No. 17 of the balancing set, when altogether we shall have

$$\left.\begin{array}{r} 100 \\ 40 \\ 1 \\ \cdot5 \\ \cdot03 \end{array}\right\} \text{in the back row,}$$

$$\left.\begin{array}{r} 1\cdot7 \\ \cdot1 \end{array}\right\} \text{in balancing coils and bridge-wire,}$$

making in all 143·33 ohms.

Should a Weston cell having an electromotive force of 1·0186 volt be substituted for the Clark, the only alteration necessary would be to short-circuit coils 40, 1, and ·5, and to move the connector from the third to the sixth of the set of hundredths. The compensating current is furnished by a small secondary cell, in series with which is a dial resistance capable of variation up to 200 ohms by steps of ·005 ohm.

The four thermojunction circuits provided are connected to a selector switch, by means of which each successively or any two of them connected in opposition may be brought into circuit, and the change-over from the standard cell connexion required in the preliminary adjustment is made at the two-way switch at the front left-hand corner, by means of which the galvanometer may be put into the circuit desired.

Constructional details are given in the paper referred to. All the coils employed are of selected manganin, carefully annealed, and all connexions are made by mercury caps and copper short-circuiting pieces, the only metals employed anywhere in the parts carrying current being copper and manganin.

The values of those coils in the box which were used in this work as determined at the conclusion of the comparison experiments are given in the following table. Any alteration in their relative values which had taken place since the first standardization is undoubtedly so small as to be quite negligible compared with other errors in thermocouple work.*

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* It is obvious that in building down to obtain a convenient standard of thermal electromotive force—100 or 1000 microvolts—so long as the *relative* values of the coils employed remain the same, their absolute value is of no moment.

A matter of great importance, however, is to measure the value of each coil in exactly the same way as it is used. Accordingly, for this standardization a potential method of measurement was employed in all cases, the current and potential leads being connected exactly as in actual work.

Coil Values in ohms at 17° C.

Coils in main row nominal value ·1 each.

| No. 1 | ·100011 | No. 6 | ·100022 | No. 11 | ·100019 | No. 16 | ·100026 |
|---|---|---|---|---|---|---|---|
| „ 2 | ·100028 | „ 7 | ·100046 | „ 12 | ·100026 | „ 17 | ·100025 |
| „ 3 | ·100025 | „ 8 | ·100026 | „ 13 | ·100023 | „ 18 | ·100022 |
| „ 4 | ·100021 | „ 9 | ·100019 | „ 14 | ·100025 | „ 19 | ·100029 |
| „ 5 | ·100022 | „ 10 | ·100022 | „ 15 | ·100020 | „ 20 | ·100033 |

Mean value of the 20 tenth-ohm coils = ·100024.

Total    „     „     slide-wire set of 11 coils shunted by slide-wire = ·100043.

Maximum variation in the resistance of slide-wire set for different positions of the slide-wire connector = ·000008.

Value of nominal 100-ohm coil = 100·029.

The auxiliary set of ·01 ohm coils for temperature compensation were all found to be within $\frac{1}{1000}$ part of their face value.

## XII. *Standard of Electromotive Force.*

The standards of E.M.F. used with this potentiometer were two similar H-form cadmium-sulphate cells with saturated solution of the type employed at the Reichsanstalt, and made up as part of a large batch of similar ones by Mr. F. E. SMITH in 1902.   From Mr. SMITH's measurements as to the relation of the E.M.F. of these cells to the standard Clark cells of the laboratory, and from other data, it is practically certain that the error committed in assuming their E.M.F. to be identical with those at the Reichsanstalt is not greater than 1 part in 10,000, which corresponds to a tenth of a degree at 1000° C. with the thermojunctions employed.   For the E.M.F. of each of these cells, which throughout the work were never found to differ by more than ·0001 volt, the Reichsanstalt official value, namely 1·0186 volt at 20° C., was assumed.

The following table gives the value of the total resistance used in the potentiometer at different temperatures to adjust the E.M.F. over each coil of the main row to exactly 1000 microvolts :—

| Temperature . . | 5°. | 10°. | 15°. | 20°. | 25°. |
|---|---|---|---|---|---|
| Resistance.   .   . | 101·90 | 101·89 | 101·88 | 101·86 | 101·84 |

The table of the values of the coils in the potentiometer shows that the actual resistance corresponding to a nominal value of 101·86 ohm is 101·889.   The relation of this to the mean value of the main set of tenths, namely ·100024 ohm, is well

within 1 part in 10,000 of its nominal value, and an inspection of the table shows that the cumulative effect of individual coil errors is very small. No corrections were therefore applied to any of the potentiometer readings.

Special precautions were taken to avoid the effect of temperature variation both on the accumulator furnishing the compensating current of ·01 ampère and on the standard cells, both being placed in double-walled boxes surrounded by a thick layer of cork clippings. Under these conditions the daily temperature range in the boxes was found to be reduced to about one-fifth of the value outside them, and the compensating current could generally be kept to within $\frac{1}{10,000}$ of its value for an hour at a time without adjustment.

## XIII. *The Thermojunctions.*

The thermojunctions used in this research were composed of pure platinum with platinum containing 10 per cent. of rhodium, and were all ·6 millim. diameter. They were obtained from HERÄUS of Hanau, through Dr. HOLBORN of the Reichsanstalt, and were compared by him with the standard junctions of the Reichsanstalt at a number of fixed points.

In a letter to the Director of the Laboratory, Dr. HOLBORN says : "Two elements were compared at four points, the melting-points of Zn, Sb, Ag, and Cu, and gave the following results* :—

|  | Temperature. | Microvolts. |
|---|---|---|
|  | ° C. |  |
| Zinc . . . . . . . | 419·0 | 340$_9$ |
| Antimony . . . . . | 630·5 | 550$_4$ |
| Silver (in graphite). . . | 961·5 | 908$_3$ |
| Copper (in air) . . . . | 1065·0 | 1027$_7$ |

Before commencing comparisons with the gas thermometer, three independent determinations of the freezing-point of silver were taken in an electrically heated crucible furnace with one of these junctions N.P.L. 2; in these experiments two different observers took part, and the three results were :—

$$\left. \begin{array}{c} 9087 \\ 9092 \\ 9082 \end{array} \right\} \text{microvolts.}$$

The mean of these, 9087, agrees very closely with the datum given above. Junction N.P.L. 2 was selected for the comparison, while N.P.L. 1 was reserved as a

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* In all experiments with thermojunctions here referred to, it is to be understood that the cold junction is at 0° C.

master standard, and was compared with No. 2 in a specially arranged electric furnace at temperatures up to 1200° C. before and after the investigation.

The results of these comparisons show conclusively that Junction No. 2 has not suffered any material alteration during its protracted heatings at high temperatures.*

The E.M.F. of the two junctions, as given by a comparison made at the close of the work, is shown in the following table :—

| Approximate temperature. | Microvolts. | | Difference, 1 − 2. | Difference in ° C. |
| --- | --- | --- | --- | --- |
| | No. 1. | No. 2. | | |
| 290 | 2202·3 | 2202·1 | + ·2 | + ·02 |
| 385 | 3089·1 | 3089·4 | − ·3 | − ·03 |
| 473 | . 3930·6 | 3930·3 | − ·3 | − ·03 |
| 489 | 4088·1 | 4084·8 | + 3·3 | + ·34 |
| 628 | 5474·3 | 5471·3 | + 3·0 | + ·30 |
| 795 | 7254·5 | 7249·3 | + 5·2 | + ·48 |
| 859 | 7937·5 | 7931·5 | + 6·0 | + ·54 |
| 1152 | 11308·5 | 11303·8 | + 4·7 | + ·39 |

## XIV. *Formulæ for Thermojunctions.*

From the values of the E.M.F. of N.P.L. 1 and 2, as determined above by HOLBORN, a formula involving two powers of the temperature was calculated by least squares to give the relation between E.M.F. and temperature to represent the Reichsanstalt's scale.

The formula

$$E_t = -304 + 8{\cdot}165\,t + 0{\cdot}001663\,t^2$$

gives residuals at the four melting-points given above much smaller than their probable error.

The corresponding formula for HOLBORN and DAY's own standard junction $T_2$, using only the values of the temperature obtained from the gas thermometer with bulb of platinum-iridium and employing the revised data for the expansion of this material at high temperatures,† is

$$E_t = -310 + 8{\cdot}048\,t + 0{\cdot}00172\,t^2.$$

The following table gives side by side the E.M.F. of HOLBORN and DAY's junction $T_2$, and of our own junctions at temperatures above 300° :—

* At the conclusion of the second set of comparisons the metallic lustre of some inches of the platinum wire was decidedly impaired, due probably to the natural disintegration of the material, but this did not appear to be accompanied by the smallest change in the E.M.F. of the junction.

† HOLBORN and DAY, 'WIED. Annalen,' 1900, vol. 2, p. 520. The change from the original formula for the expansion of the bulb involves a correction of the scale amounting to 4° at 1000°.

| T. | NPL 1 and 2. | | Holborn and Day's $T_2$. | | Difference NPL − $T_2$. |
|---|---|---|---|---|---|
| 300 | 2295 | | 2260 | | 35 |
| | | 933 | | 925 | |
| 400 | 3228 | | 3185 | | 43 |
| | | 966 | | 960 | |
| 500 | 4194 | | 4145 | | 49 |
| | | 1000 | | 994 | |
| 600 | 5194 | | 5139 | | 55 |
| | | 1032 | | 1029 | |
| 700 | 6226 | | 6168 | | 58 |
| | | 1066 | | 1063 | |
| 800 | 7292 | | 7231 | | 61 |
| | | 1099 | | 1097 | |
| 900 | 8391 | | 8328 | | 63 |
| | | 1133 | | 1132 | |
| 1000 | 9524 | | 9460 | | 64 |
| | | 1166 | | 1166 | |
| 1100 | 10690 | | 10626 | | 64 |

It will be observed that both the formulæ just quoted do not apply at lower temperatures, being nearly 40° C. in error at 0°, and that therefore extrapolation downwards even over a narrow range is not permissible. The error of the formula for NPL 2 was determined to be 4° at 200°.

## XV. *Determination of the Fixed Points* 0° *and* 100° *and Sulphur Boiling-point.*

The determinations of the fixed points 0° and 100° for the gas and platinum thermometers were made in baths specially built for each kind of instrument. The ice-points were taken in glass vessels of a capacity of about 6 and 3 litres respectively, consisting each of an inverted glass bell-jar with draining arrangement below, and surrounded by a thick packing of cork clippings. Very little melting of ice took place even in 12 hours, the upper surface of the ice being protected by a thick felt covering wrapped round the stem of the instrument.

The block-ice previous to use was always well washed and finely divided by an ice plane, and was repeatedly tested for dissolved impurity, the method adopted being to ascertain the amount of chlorine present by addition of silver nitrate to the drainings. It was found satisfactory, except on one occasion.

The steam-point apparatus for the gas thermometer was of the usual type with concentric tubes, but was arranged to be easily changed from the vertical to the horizontal position by suitable couplings of wide compo tubing, connecting it to the boiler. The steam- and zero-points for any series of comparisons were always taken with the reservoir in the position in which it was used in that series, and in all cases the amount of stem emergent was made as nearly as possible the same as in the comparisons. The steam bath as arranged for the horizontal position is shown in

fig. 6. The bulb of the gas thermometer, resting on a small piece of cork, occupies the centre of the inner tube, through which steam brought direct from the boiler by a well protected wide tube is circulated. The steam issuing from the outer jacket is
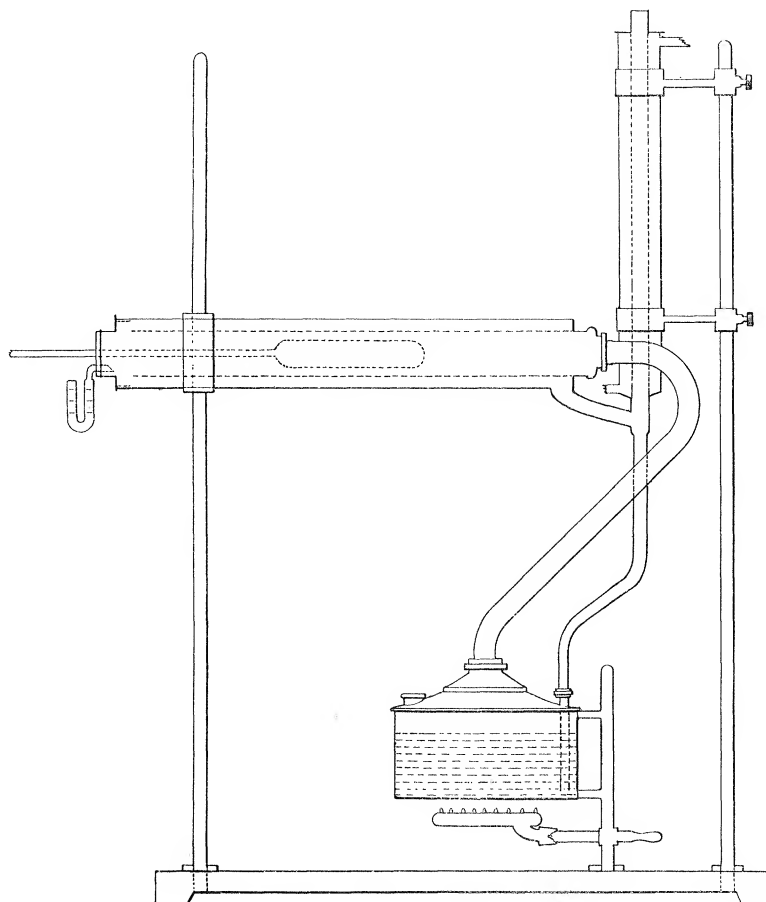


Fig. 6. Steam bath.

condensed and returned to the boiler as shown. The excess of the steam pressure over that of the atmosphere, which in these experiments was seldom over 1 millim. of water, is indicated by a small graduated water gauge.

The sulphur-points of the platinum thermometers were taken in the usual manner in the well-known Callendar form of boiling-point apparatus. The only departure from previous custom being that for the glass boiling tube was substituted one of thin weldless steel,* which is more durable and can be heated up quickly without being removed from its asbestos cover. Careful comparisons of this form with the older glass apparatus showed no systematic discrepancy.

For the sulphur boiling-point CALLENDAR's old value 444·53° C. at normal pressure

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* An ordinary iron tube such as gas or steam pipe cannot be used for this purpose, as owing to conduction from the flame of the burner upwards there is a tendency for the sulphur vapour to become superheated.

was taken. The sulphur-points were always taken on days when the pressure was not far removed from 760 millims. to eliminate the uncertainty as to the co-efficient to use when reducing to normal pressure.

## XVI. *Electric Furnaces.*

Two different electric furnaces were employed in this work. Their dimensions were similar, but they differed in that in the first the heating-wire was wound uniformly, and in the second an approximation to a logarithmic spiral was made at each end, the turns being gradually crowded, so that the cooling effect of the ends was in a great measure compensated by the additional heat supply. Both furnaces were wound with wire of pure nickel about 1·6 millims. diameter. The heating current was supplied from a special battery of 56 accumulators reserved for this purpose, which was divided into four groups of 14 cells, capable of being coupled in series or parallel, as desired. A set of large well-ventilated manganin resistances, formed of two No. 9 wires in parallel, and capable of carrying 100 ampères without undue heat-ing, was arranged so that the external resistance of the circuit could be altered by steps of ·025 ohm up to 3·2 ohms, thus enabling any desired amount of energy to be put into the furnace at will. The con-struction of the furnace and the disposition of the different instruments within it is shown in fig. 7. The nickel heating-wire is wound upon the inner tube of unglazed biscuit porcelain, and in order to prevent the turns becoming short-circuited when hot, the whole of the wound portion is covered with a thin layer of " purimachos" which is baked on at a moderate heat. The leading-in wires are doubled or trebled in all cases. The bulb of the gas thermo-meter is supported on a small bridge of fire-clay resting on the furnace bottom, and the standard platinum thermometer and thermojunction are arranged as shown, great care being taken that neither the wires of the junction nor the thin porcelain
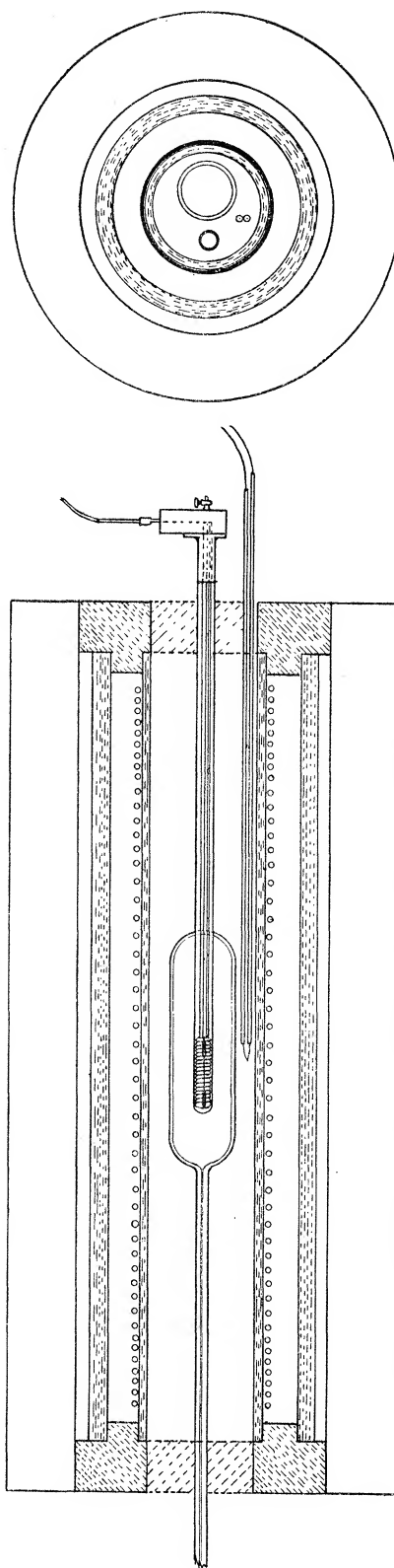


Fig. 7. Electric furnace, longitudinal and transverse sections.

capillary tubes used to cover them shall anywhere touch the furnace wall. As previous experience with gas and platinum thermometers, whose walls were of porcelain, had shown how very much more slowly the transfer of heat took place through this material than through metal or glass, even when surrounded by a stirred liquid, it was judged preferable to make a comparatively small number of high-temperature experiments, in which great constancy of temperature was attained for some time previous to and during the observations, rather than to attempt to obtain mean values from more extended series under less perfect conditions. With this object, a Callendar recorder was connected to a second platinum thermometer placed in the furnace, and, during the adjustment of the temperature and the comparisons, records from this instrument were taken on an open scale. The use of the recorder greatly shortened the time necessary for the establishment of a steady temperature by guiding the observer as to the manipulation of the resistances in the heating circuit. In addition to the large set of resistances in the heating circuit, a set of coils of ·01 ohm each was placed close to the recorder, and it was found that a change of one step on this set made all the difference between a steady state and a gradual rise or fall in the temperature of the furnace when equilibrium had been nearly established. When desired, it was quite easy to keep the furnace temperature constant to about a fifth of a degree for half-an-hour at a time, at temperatures as high as 1000° C., but in most of the experiments the temperature was intentionally allowed to rise very slowly. Without these precautions, comparisons between instruments of such widely differing " lag " as a bare thermojunction and a gas thermometer with porcelain reservoir, whose walls were 2 millims. in thickness, would have undoubtedly been liable to serious error.

### XVII. *Furnace Correction.*

In order to investigate the distribution of temperature throughout the space filled by the gas thermometer bulb, which was about 130 millims. in length, a pair of thermojunctions, quite independent of the standard, were arranged so as to measure the temperature difference between the centre of the furnace and points further out, and so obtain a correction to be applied to the readings of the gas thermometer, to reduce its indications to what it would have registered had the whole of the bulb been at the same temperature as the middle point.

### XVIII. *Exploration of Furnace.*

For this purpose a thin wire junction of platinum with platinum iridium was chosen on account of its great sensitiveness at high temperatures. This was made up to work differentially, and was composed of a piece of platinum iridium between two pieces of platinum, thus forming two junctions, which were both placed in the hot space, entering the furnace from opposite ends. The wires were stiffened by threading

them through thin porcelain capillary tubes, and the two junctions of the platinum with the copper forming the rest of the circuit were placed together in ice.

The E.M.F. given by this element for a difference of 1° between its two hot junctions is given in the table below.* This is obtained by direct comparison over the whole range of a simple junction made up of wires from the same reels, which had been similarly treated.

| Temperature in °C. | Difference for 1° in microvolts. |
| --- | --- |
| 500 | 16·4 |
| 600 | 16·6 |
| 700 | 16·8 |
| 800 | 16·9 |
| 900 | 17·1 |
| 1000 | 17·2 |

One of the junctions was carefully placed and kept at the middle point of the gas thermometer bulb, and the second was arranged so that it could be pushed backward and forward into positions 2, 4, 6, and 8 centims. to the right and left of this point, observations being made of the differential E.M.F. produced in each of these positions.

From a number of such observations made after the different comparisons and spaced over the interval 400° C. to 1000° C., curves were constructed showing the distribution of temperature over this space for each of the furnaces used, and by measuring these curves the difference between the average temperature of the bulb and that of the central point, where the standard thermojunction was placed, could easily be found.

For the compensated furnace the mechanical centre was found not to quite coincide with the position of highest temperature at the higher ranges, though over the lower part of the scale the curves were practically symmetrical.

The corrections obtained by this method from the different explorations were plotted as ordinates against furnace temperatures as abscissæ, and from the mean curve thus obtained the following values were deduced as the mean corrections at different temperatures :—

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* In practice, when the junctions were placed as close together in the furnace as possible without actual contact, owing to the inevitable small secondary effects arising from unsymmetric heating of the junction wires, the E.M.F. round the circuit was not always zero. When, however, the junctions were so supported as to nowhere touch the furnace wall, the total effect was always quite small and was allowed for in each case.

| Furnace temperature. | Corrections to gas thermometer reading for compensated furnace experiments. |
|:---:|:---:|
| ° C. | ° C. |
| 500 | + 1·7 |
| 600 | + 1·5 |
| 700 | + 1·3 |
| 800 | + 1·1 |
| 900 | + ·8 |
| 1000 | + ·0 |

From this table the numbers in the column headed " furnace correction " in Series II. and III. have been deduced by interpolation. For the uncompensated furnace used in the earlier experiments the correction is uncertain to about $0·5°$ C., and the results are, therefore, only given to this accuracy in the final column representing the " corrected gas thermometer temperature."

## XIX. *Method of Calculation.*

The observations taken with the gas thermometer were calculated according to the usual formula.

Let $H_0$ = pressure at $0°$,

$H$ =    „     „    $T°$,

and let $t_1$ and $t'_1$ be temperatures of " dead space " with bulb at $0°$ and at temperature $T°$.

Let $\dfrac{v}{V_0}$ = relation of the volume of the whole " dead space " to the volume of the bulb at $0°$,

$\alpha$ = coefficient of expansion of the gas at constant volume between $0°$ and $100°$,

$3\beta$ = the mean cubical coefficient of expansion of the porcelain bulb between $0°$ and $T°$ ;

then

$$H_0\left(1 + \frac{v}{V_0}\frac{1}{1 + \alpha t_1}\right) = \frac{H}{1 + \alpha T}\left\{1 + 3\beta T + \frac{v}{V_0}\left(\frac{1 + \alpha T}{1 + \alpha t'_1}\right)\right\},$$

therefore

$$T = \frac{H}{\alpha H_0\left(1 + \dfrac{v}{V}\dfrac{1}{1 + \alpha t_1}\right)}\left\{1 + 3\beta T + \frac{v}{V_0}\left(\frac{1 + \alpha T}{1 + \alpha t'_1}\right)\right\} - \frac{1}{\alpha}.$$

For a given filling $H_0\left(1 + \dfrac{v}{V_0}\dfrac{1}{1 + \alpha t_1}\right)$ is a constant if the zero-point remains constant.

The values of $\alpha$ obtained from the steam-point determinations were calculated from the same formula, inserting the value of T obtained from the Regnault-Broch table for the boiling-point of water under the prevailing pressure, as given in the example.

## XX. *Series II.*

### ICE-POINT; 3.  April 16, 1903.

*Observer, J. A. H.*

| Time. | Scale reading. | Dead space.* (1.) | Mercury columns. (2.) | (3.) | Barometer. | Barometer temperature. |
|---|---|---|---|---|---|---|
| h.  m. | | | | | | |
| 2   20 | 159·86 | 13·8 | 14·2 | 14·0 | 768·73 | 15·0 |
| 23 | 159·98 | 13·8 | 14·2 | 14·0 | ·62 | 15·0 |
| 26 | 160·00 | 13·9 | 14·3 | 14·05 | ·58 | 15·2 |
| 30 | 160·00 | 14·0 | 14·3 | 14·15 | ·60 | 15·2 |
| | 159·96 | 13·9 | 14·25 | 14·05 | 768·63 | 15·1 |
| Index corr. = | 747·18 | − ·0 | − ·00 | − ·00 | Lat. corr. = + ·43 | − ·1 |
| | | | | | Temp. corr. = − 1·88 | |
| | | | | | Scale corr. = − ·05 | |
| | 587·22 | 13·9 | 14·25 | 14·05 | | 15·0 |
| Temp. corr. = − 1·35 | | | | | | |
| Lat. corr. = + 0·32 | | Merc. col. temp. } 14·15° | | | Corr. bar. = 767·13 | |
| | 586·19 | | | | | |
| | 767·13 | | | | | |
| $H_0$ = | 180·94 | | | | | |

It will be observed that during the 10 minutes covered by the observations the barometric pressure fell rather more than 0·1 millim. while the scale reading rose a corresponding amount.

* On this occasion the variations in the temperature of the jacket water were so abnormally great during the preliminary period before the experiment, that it was decided to dispense with the circulation in the space round the platinum capillary and take the whole dead space as being at the temperature of the part below the stopper in the closed limb of the manometer, given by thermometer No. 1.

STEAM-POINT; 3.    April 16, 1903.

*Observer, J. A. H.*

| Time. | Scale reading. | Dead space. (1.) | Mercury columns. | | Barometer. | Barometer temperature. |
|---|---|---|---|---|---|---|
| | | | (2.) | (3.) | | |
| h.  m. | | | | | | |
| 3   15 | 225·58 | 14·2 | 14·55 | 14·30 | 768·60 | 15·5 |
| 22 | ·63 | 14·2 | 14·65 | 14·35 | ·62 | 15·5 |
| 30 | ·67 | 14·35 | 14·70 | 14·50 | ·60 | 15·5 |
| 32 | ·58 | 14·45 | 14·70 | 14·50 | ·55 | 15·5 |

225·61    14·3    14·65    14·4                768·59    15·5
747·18  − ·0    − ·0    − ·0    Lat. corr.  = +    ·43  − ·1
                                    Temp. corr. =  −  1·94
521·57    14·3    14·65    14·4    Scale corr. =  −    ·05    15·4
Temp. corr. = − 1·23
Lat. corr.  = +   29    Col. temp. } taken as    14·5°    Corr. bar.  =  767·03

520·63
767·03

                                              millim.        millims.
H =    246·40    Corr. bar. . . . . . . . =        767·03
                Water manometer on steam bath =    +1    =  +    ·07

                Total pressure on steam . . .  =        767·10

                Boiling-point of water at 767·10 millims.  = 100·26°.

                                α = ·003669₁.

## XXI. *Accuracy of Gas Thermometer Constant Determinations.*

As an example of the kind of accuracy attained in the determination of the constants of the gas thermometer, the individual values of the zero- and steam-points taken before the comparisons in Series II. is given along with the readings taken in one complete determination of each.

For calculation of experiments of this kind, where no gradual systematic drift is expected, instead of utilising for calculating the individual values of the α the single determinations of the ice-point, the mean of the ice-points is taken for this purpose.

The difference between the value of α found before and after the comparisons is within the limits of experimental error. It will be seen from the table appended, which gives in an abridged form the calculation of the gas thermometer temperatures in the 13 experiments of Series II., that the difference has, however, been treated as real, and assumed to vary with the time. A more serious change has, however, taken

place in the quantity of gas present in the bulb. The change in this case has also been assumed as proportional to the time, since before the first experiment was made the whole furnace was maintained at a high temperature for some time, and there is no sufficient evidence that the change occurred otherwise than regularly. So far as could be found by testing on the pump after the conclusion of the series no leak could be discovered, even when the bulb was at a full red heat.

### Constants for Gas Thermometer. Series II.

I. Before the comparisons :—

#### Ice-points.

|  | $H_0$. | Dead space temperature. |
|---|---|---|
| (1) | 180·90 | 12·6 |
| (2) | ·93 | 13·1 |
| (3) | ·94 | 13·9 |
| Mean ice-point before comparison. $=$ | 180·923 | |

#### Steam-points.

|  | $H_{100}$. | Dead space temperature. | $\alpha$. |
|---|---|---|---|
| (1) | 246·24 | 13·95 | ·003669$_9$ |
| (2) | ·26 | 13·75 | ·003671$_1$ |
| (3) | ·23 | 14·3 | ·003669$_1$ |
| Mean value of $\alpha$ before comparisons $=$ | | | ·003670$_0$ |

II. After comparisons :—

#### Ice-points.

|  | $H_0$. | Dead space temperature. |
|---|---|---|
| (4) | 182·67 | 17·4 |
| (5) | 182·65 | 17·8 |
| Mean ice-point after comparisons. $=$ | 182·66 | |

#### Steam-points.

|  | $H_{100}$. | Dead space temperature. | $\alpha$. |
|---|---|---|---|
| (4) | 248·64 | 16·3 | ·003671$_5$ |
| (5) | 248·63 | 16·6 | ·003670$_8$ |
| | | | ·003671$_2$ |

XXII. *Summary of Calculation of Gas Thermometer Temperatures.*
Series II.

| No. of experiment. | Scale reading. | Index correction. | Differential pressure. | Mean temperature of manometer column. | Corrected differential pressure. | Barometer reading. | Barometer temperature. | Corrected barometer reading. | Mean dead-space temperature. | H. | $H_0\left[1 + \dfrac{v}{V}\left(\dfrac{1}{1+\alpha t_1}\right)\right]$. | $\alpha \cdot 00$. | Gas thermometer reading. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 552·43 | 747·18 | −194·75 | 15·8 | −194·36 | 758·62 | 15·7 | 757·05 | 14·4 | 562·67 | 182·636 | 3670 | 597·9 |
| 2 | 664·01 | 747·18 | −83·17 | 16·3 | −83·00 | 757·23 | 16·2 | 755·60 | 15·4 | 672·60 | 182·781 | 3670 | 775·6 |
| 3 | 581·92 | 747·18 | −165·26 | 12·8 | −165·01 | 755·55 | 11·9 | 754·45 | 12·7 | 589·44 | 182·926 | 3670 | 639·7 |
| 4 | 612·32 | 747·18 | −134·86 | 13·5 | −134·63 | 751·36 | 12·8 | 750·15 | 13·6 | 615·52 | 183·070 | 3670 | 681·1 |
| 5 | 515·34 | 747·18 | −231·84 | 15·1 | −231·41 | 749·70 | 15·3 | 748·20 | 15·0 | 516·79 | 183·216 | 3670₅ | 521·4 |
| 6 | 734·00 | 747·18 | −13·18 | 16·6 | −13·16 | 749·72 | 16·45 | 748·08 | 15·6 | 734·92 | 183·361 | 3670₅ | 874·2 |
| 7 | 786·11 | 747·18 | +38·93 | 17·3 | +38·84 | 750·00 | 17·0 | 748·28 | 16·5 | 787·12 | 183·505 | 3670₅ | 959·5 |
| 8 | 562·43 | 747·18 | −184·75 | 16·4 | −184·36 | 751·40 | 16·55 | 749·75 | 16·3 | 565·39 | 183·650 | 3670₅ | 597·0 |
| 9 | 699·39 | 747·18 | −47·79 | 15·1 | −47·71 | 752·08 | 14·4 | 750·68 | 15·0 | 702·97 | 183·795 | 3670₅ | 819·0 |
| 10 | 540·90 | 747·18 | −206·28 | 15·9 | −205·87 | 754·96 | 15·95 | 753·37 | 15·7 | 547·50 | 183·940 | 3671 | 566·9 |
| 11 | 711·16 | 747·18 | −36·02 | 15·5 | −35·95 | 748·52 | 14·3 | 747·14 | 15·4 | 711·19 | 184·085 | 3671 | 830·4 |
| 12 | 733·52 | 747·18 | −13·66 | 14·6 | −13·64 | 748·17 | 13·8 | 746·86 | 14·4 | 733·22 | 184·230 | 3671 | 865·8 |
| 13 | 816·95 | 747·18 | +69·77 | 14·9 | +69·64 | 749·92 | 13·9 | 748·59 | 14·8 | 818·23 | 184·375 | 3671 | 1005·0 |

The "index correction" was determined on two occasions during this series, the individual readings being:—

$$747 \cdot 16$$
$$747 \cdot 19$$
$$747 \cdot 18$$
$$747 \cdot 19$$
$$747 \cdot 17$$

Mean $= \overline{747 \cdot 18}$

The relation $\dfrac{v}{V} = \cdot 010_0$.

## XXIII. *Method of calculating Comparison Experiment.*

As an example of the method employed in making and calculating out an experiment, the results of the observations made in Experiment No. 13 of Series II. are given in full. During this experiment the steadiness of the temperature of the furnace was perhaps a little above the average, but the room temperature and that of the resistance-boxes and of the various mercury columns of the gas thermometer and barometer were rising more rapidly than usual. According to observations taken on the thermojunctions and platinum thermometer, the steady state had been maintained for about 10 minutes before the first recorded reading on the gas thermometer was taken. As only two observers were available, one of these took alternate observations on the gas thermometer and thermojunctions, while the second took the platinum thermometer readings.

By graphic interpolation the mean thermojunction readings, corresponding to the times at which the other instruments were read, were obtained and are given in the example.*

The readings of the gas thermometer are made as independent of one another as possible by lowering the mercury each time before a setting, raising it again slowly so as to make a new meniscus.

The calculation to the accuracy here necessary of the air temperature corresponding to definite platinum temperatures is somewhat laborious, if the formula

$$T = \left(\frac{5000}{\delta} + 50\right) - \sqrt{\left(\frac{5000}{\delta} + 50\right)^2 - \frac{10,000\,pt}{\delta}}$$

has to be applied for each observation. The graphic methods used by HEYCOCK and NEVILLE cannot easily be made sufficiently accurate. Since the value of $\delta$ for pure platinum wire has been found to be 1·5, varying from one specimen to another within very narrow limits, the most suitable method of effecting this conversion was found to be to construct a table giving, for a sufficient number of points, the value of T for given values of $pt$ when $\delta = 1·5$. A second table gives the correction to apply to the value of T thus obtained, if the $\delta$ differs from the standard value by a small amount.

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* The letters AB, BA refer to the position of the reversing switches leading to the potentiometer. Owing to small Thomson and Peltier effects, there was generally a small difference between the two positions. In this case it is rather above the average. There was no difficulty in setting to ·1 microvolt, so the readings are given to this figure, though it is not considered as having any significance in temperature measurement.

GAS Thermometer Readings.　Expt. 13, May 1, 1903.　*Observer, J. A. H.*

| Time. | Scale. | Thermometers. | | | | Barometer. | Barometer temperature. |
|---|---|---|---|---|---|---|---|
| | | Dead space below point. | Columns. | | Water circulation. | | |
| | | (1.) | (2.) | (3.) | (4.) | | |
| h. m. s. | | | | | | | |
| 5　5　0 | 816·86 | 14·7 | 14·90 | 14·75 | 15·2 | 749·85 | 14·0 |
| 　9　0 | 816·96 | ·8 | ·95 | ·8 | ·2 | ·90 | ·0 |
| 14　0 | 817·00 | ·95 | 15·00 | ·9 | ·1 | ·95 | ·0 |
| 17　30 | 817·00 | 15·0 | ·05 | ·95 | ·1 | 750·00 | ·0 |
| | 816·95 | 14·85 | 14·95 | 14·85 | 15·15 | 749·92 | 14·0 |
| Index corr. = | − 747·18 | − ·00 | − ·00 | − ·00 | − ·40 | Scale corr. = − ·05 | ·1 |
| | | | | | | Lat. corr. − + ·42 | |
| | + 69·77 | 14·85 | 14·95 | 14·85 | 14·75 | Temp. corr. − 1·70 | 13·9 |
| Lat. corr. = | + ·04 | | | | | | |
| Temp. corr. = | − ·17 | | | | | Corr. bar. = 748·59 | |
| | + 69·64 | | | | | | |
| | 748·59 | | | | | | |
| H = | 818·23 | | | | | | |

Mean temperature of dead space $= 14\cdot80 = t_1'$.

$$H_0\left(1 + \frac{v}{V}\cdot\frac{1}{1 + at_1}\right) = 184\cdot375. \qquad \frac{v}{V} = \cdot010_0.$$

$$a = \cdot003671_0.$$

$$T_{gas} = 1005\cdot2°. \qquad (1 + 3\beta T) = 1\cdot01230,$$

where T taken from junction $= 1004\cdot3°$.

A second approximation, employing 1005·2° in the dilatation term, makes the final gas thermometer temperature 1005·0° C.

THERMOJUNCTION Readings.　Expt. 13.

*Observer, J. A. H.*

| Time. | Microvolts. | Cell resistance and temperature. | Series resistance. |
|---|---|---|---|
| h. m. s. | | | |
| 5　0　0 | 9570·5 AB | 101·89 | 103·036 |
| 　2　0 | 75·8 BA | T = 13·7 | — |
| 　7　30 | 76·1 BA | — | — |
| 　8　15 | 73·0 AB | — | — |
| 11　30 | 73·5 AB | — | 103·036 |
| 13　0 | 72·0 AB | — | — |
| 16　0 | 71·2 AB | — | — |
| 16　30 | 75·2 BA | — | — |
| 20　40 | 71·2 AB | — | — |
| 21　0 | 75·4 BA | — | 103·036 |

From these observations plotted, the following were deduced as the thermojunction readings simultaneous with the gas thermometer readings :—

| Time. | Mean microvolts. |
|---|---|
| h.  m.  s.<br>5  5  0 | 9573·8 |
| 9  0 | 9574·8 |
| 14  0 | 9573·9 |
| 17  30 | 9573·2 |
| | Mean = 9573·9 |

From table of E.M.F. of junctions the value of the E.M.F. for 1000° C. = 9524·0, and the difference for $10° = 115·1$ microvolts.

Whence the temperature corresponding to $9573·9 = 1004·3°$

$$T \text{ (thermojunction)} = 1004·3.$$

PLATINUM Thermometer Readings.    Expt. 13.

*Observer, W. H.*

| Time. | Coils. | Bridge wire. | Box temperature. | Centre = −0·29. |
|---|---|---|---|---|
| h.  m.  s.<br>5  5  0 | ABEFGI | − 2·530 | 14·32 | — |
| 9  0 | — | − 2·660 | ·38 | — |
| 14  0 | — | − 2·686 | ·44 | — |
| 17  30 | — | − 2·784 | ·48 | — |
| | 1129·454 | − 2·665 | 14·40 | — |

| R. | R − R$_0$. | *pt.* | T ($\delta = 1·50$). | Difference for ·009 in $\delta$. |
|---|---|---|---|---|
| 1125·44 | 868·79 | 868·70 | 1005·18 | − ·82 |

$$T (\delta = 1·491) = 1004·37°.$$

Summary .—

| | | |
|---|---|---|
| T gas (found) | = 1005·0 |
| Furnace correction | = + 0·0 |
| T gas corrected | = 1005·0 |
| T thermojunction | = 1004·4 |
| T platinum thermometer | = 1004·37 |

## XXIV. *First Set of Comparisons.*     October, 1902.

<div align="right">millims.</div>

Mean ice-point of gas thermometer before observations . . . 336·41

,,          ,,          ,,     after     ,,        . . . 336·25

Mean value of $\alpha$ before $= \cdot003675$,

,,          ,,     after $= \cdot003676$.

$$\frac{v}{V_0} = \cdot008_3.$$

Platinum thermometer $BA_2$:—

$R_0 = 2\cdot57453$ ohms before commencing,

$= 2\cdot57465$   ,,   after seventh experiment,

$= 2\cdot57433$   ,,   at conclusion,

F.I. $= 1\cdot00034$ before commencing,

$= 1\cdot00020$ at end,

$\delta = 1\cdot510$   mean of all observations.

FIRST Set of Comparisons.   Furnace wound uniformly.   $BA_2$ and Gas Thermometer.

| No. of experiment. | *pt.* | $T (\delta = 1\cdot50)$ from Table I. | $T (\delta = 1\cdot491)$ from Table II. | Gas thermometer reading. | Compensation correction. | Corrected gas thermometer reading. |
|---|---|---|---|---|---|---|
| 1 | 482·54 | 514·66 | 514·47 | 510·2 | +5 | 515 |
| 2 | 604·96 | 660·48 | 660·15 | 655·2 | +4 | 659 |
| 3 | 606·48 | 662·35 | 662·02 | 659·1 | +4 | 663 |
| 4 | 610·02 | 666·69 | 666·36 | 661·7 | +4 | 665$\frac{1}{2}$ |
| 5 | 459·21 | 487·66 | 487·49 | 483·4 | +5 | 488$\frac{1}{2}$ |
| 6 | 749·07 | 843·03 | 842·46 | 839·7 | +3 | 842$\frac{1}{2}$ |
| 7 | 750·24 | 844·56 | 843·99 | 840·7 | +3 | 843$\frac{1}{2}$ |
| 8 | 481·53 | 513·36 | 513·17 | 509·5 | +5 | 514$\frac{1}{2}$ |
| 9 | 394·96 | 414·52 | 414·40 | 411·2 | +5$\frac{1}{2}$ | 416$\frac{1}{2}$ |

RESULTS of Comparison arranged in Order of Ascending Temperature.

| | T from platinum. | T from gas corrected. | Difference. T gas – T platinum. |
|---|---|---|---|
| 9 | 414·4 | 416$\frac{1}{2}$ | +2 |
| 5 | 487·5 | 488$\frac{7}{2}$ | +1 |
| 8 | 513·2 | 514$\frac{1}{2}$ | +1 |
| 1 | 514·5 | 515 | + ·5 |
| 2 | 660·1 | 659 | – 1 |
| 3 | 662·0 | 663 | +1 |
| 4 | 666·4 | 665$\frac{1}{2}$ | – 1 |
| 6 | 842·5 | 842$\frac{1}{2}$ | +0 |
| 7 | 844·0 | 843$\frac{1}{2}$ | – ·5 |

*Second Set of Comparisons.* April and May, 1903.

millims.

Mean ice-point of gas thermometer before comparisons . . . . $180\cdot92_3$

,,       ,,       ,,     after       ,,    . . . . $182\cdot66$

Mean value of $\alpha$ from determinations before comparisons $= \cdot003670$,

,,       ,,       ,,     after       ,,     $= \cdot003671$.

$$\frac{v}{V_0} = \cdot010_0.$$

In this case there is a change in the zero-point greater than that observed in the first set and in the opposite direction. In allowing for this, the change was supposed to have been proportional to the time.

Platinum thermometer $BA_2$ :—

$R_0 = 2\cdot57251$ ohms before commencing comparisons,

    $= 2\cdot57198$    ,,    at conclusion of        ,,

$F.I. = 1\cdot00008$ before commencing       ,,

     $= 1\cdot00000$ at conclusion of       ,,

$\delta = 1\cdot491$    mean of large number of observations extending over six months.

The platinum thermometer constants of this set are expressed in terms of a slightly different unit from that employed in first set six months earlier.

SECOND Series of Comparisons; Compensated Furnace.

Gas Thermometer, Platinum Thermometer BA$_2$, and Thermojunction NPL 2.

The comparisons are for the sake of reference arranged in order of ascending temperature.

| No. of experiment. | Gas thermometer. | | | Thermojunction. | | Platinum thermometer. | | | Differences. | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Reading of gas thermometer. | Furnace correction. | Corrected temperature from gas thermometer. | Microvolts of junction. | Temperature from junction. | pt. | T, CALLENDAR's formula, $\delta=1\cdot50$ from Table I. | Temperature from platinum thermometer, $\delta=1\cdot491$ from Table II. | Gas–platinum. | Gas thermometer. | Platinum thermometer. |
| 5 | 521·4 | +1·7 | 523·1 | 4434 | 524·3 | 491·18 | 524·59 | 524·39 | −1·3 | −1·2 | +·1 |
| 10 | 566·9 | +1·6 | 568·5 | 4885 | 569·5 | 529·44 | 569·59 | 569·35 | −0·8 | −1·0 | −·2 |
| 8 | 597·0 | +1·5 | 598·5 | 5172 | 597·8 | 553·22 | 597·89 | 597·62 | +·9 | +0·7 | −·2 |
| 1 | 597·9 | +1·5 | 599·4 | 5183 | 599·0 | 554·21 | 599·07 | 598·80 | +·6 | +0·4 | −·2 |
| 3 | 639·7 | +1·4 | 641·1 | 5614 | 641·1 | 589·03 | 641·06 | 640·75 | +·4 | +0·0 | −·4 |
| 4 | 681·1 | +1·3 | 682·4 | 6048 | 683·0 | 623·20 | 682·90 | 682·54 | −·1 | −0·6 | −·5 |
| 2 | 775·6 | +1·1 | 776·7 | 7028 | 775·5 | 696·97 | 775·60 | 775·13 | +1·6 | +1·2 | −·4 |
| 9 | 819·0 | +1·0 | 820·0 | 7492 | 818·4 | 730·55 | 818·84 | 818·31 | +1·7 | +1·6 | −·1 |
| 11 | 830·4 | +1·0 | 831·4 | 7643 | 832·2 | 740·94 | 832·41 | 831·86 | −·5 | −0·8 | −·3 |
| 12 | 865·8 | +·9 | 866·7 | 8040 | 868·4 | 768·68 | 868·89 | 868·28 | −1·6 | −1·7 | −·1 |
| 6 | 874·2 | +·8 | 875·0 | 8117 | 875·4 | 773·92 | 875·86 | 875·24 | −·2 | −0·4 | −·2 |
| 7 | 959·5 | +·3 | 959·8 | 9022 | 956·0 | 833·40 | 956·21 | 955·47 | +4·3 | +3·8 | −·5 |
| 13 | 1005·0 | +·0 | 1005·0 | 9574 | 1004·4 | 868·70 | 1005·19 | 1004·37 | +·6 | +0·6 | −·0 |

*Third Set of Comparisons.* May and June, 1903.

millims.

Mean ice-point of gas thermometer before comparison $= 176\cdot28$

,,　　　　,,　　　　,,　　after　　,,　　$= 176\cdot19$

Mean value of $\alpha$ from determinations before $= \cdot003660$

,,　　　　　,,　　　　　,,　　after　$= \cdot003661$

Platinum thermometer $K_2$ :—

$R_0 = 2\cdot62864$ ohms before commencing comparisons,

$= 2\cdot62705$　,,　at conclusion of　　　　,,

F.I. $= 1\cdot00789$ before commencing　　　　,,

$= 1\cdot00819$ at conclusion of　　　　,,

$\delta = 1\cdot510$　mean value over a long period.

THIRD Series of Comparisons in Compensated Furnace.

Gas Thermometer, Platinum Thermometer K₂, and Thermojunction NPL 2.

| No. of Experiment. | Gas thermometer. | | | Thermojunction. | | Platinum thermometer. | | | Differences. | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Reading of gas thermometer. | Furnace correction. | Corrected temperature from gas thermometer. | Microvolts of junction. | Temperature from junction. | pt. | T, δ = 1·50 from Table I. | T, δ = 1·51 from Table II. | Gas–platinum. | Gas thermometer. | Platinum thermometer. |
| 2 | 451·8 | +1·9 | 453·7 | 3750 | 454·4 | 429·30 | 453·33 | 453·49 | + ·2 | − ·7 | − ·9 |
| 6 | 462·7 | +1·9 | 464·6 | 3862 | 465·9 | 438·98 | 464·36 | 464·53 | + ·1 | −1·3 | −1·4 |
| 1 | 537·6 | +1·7 | 539·3 | 4597 | 540·7 | 503·92 | 539·48 | 539·72 | − ·4 | −1·4 | −1·0 |
| 3 | 559·2 | +1·6 | 560·8 | 4814 | 562·5 | 522·14 | 560·93 | 561·19 | + ·4 | −1·7 | −1·3 |
| 5 | 641·2 | +1·4 | 642·6 | 5648 | 644·4 | 590·21 | 642·49 | 642·83 | − ·2 | −1·8 | −1·6 |
| 4 | 675·5 | +1·4 | 676·9 | 6001 | 678·5 | 618·04 | 676·56 | 676·94 | + ·0 | −1·6 | −1·6 |
| 7 | 752·6 | +1·2 | 753·8 | 6829 | 756·9 | 680·47 | 754·56 | 755·05 | −1·2 | −3·1 | −1·9 |
| 9 | 844·1 | + ·9 | 845·0 | 7779 | 844·6 | 748·18 | 841·87 | 842·49 | +2·5 | + ·4 | −2·1 |
| 8 | 864·6 | + ·9 | 865·5 | 8011 | 865·6 | 764·39 | 863·23 | 863·89 | +1·6 | − ·1 | −1·7 |
| 11 | 936·0 | + ·4 | 936·4 | 8757 | 932·5 | 813·70 | 929·30 | 930·07 | +6·3 | +3·9 | −2·4 |
| 10 | 969·8 | + ·2 | 970·0 | 9165 | 968·6 | 839·78 | 964·98 | 965·82 | +4·2 | +1·4 | −2·8 |

## XXV. *Conclusions.*

These three sets of experiments show that :—

(1.) The readings of platinum thermometers $BA_2$ and $K_2$, which may be taken as representative samples of pure platinum, when reduced to the air-scale by CALLENDAR's formula, employing his value for the boiling-point of sulphur, are in reasonably close agreement with the results given by the constant-volume nitrogen thermometer employing chemical nitrogen under low initial pressure and using the revised values for the dilatation of porcelain. The divergence of the two scales only exceeds the probable error at the higher part of the range.

(2.) The platinum thermometers, and the thermojunctions representing the temperature scale of the Reichsanstalt, based on measurements made with the gas thermometer with bulb of platinum iridium, agree very closely, the thermojunction giving apparently a slightly higher value throughout the range covered by the experiments.

As the result of these comparisons is to justify the application of CALLENDAR's parabolic formula up to $1000°$ C., the tables previously alluded to for deducing the value of T from $pt$ over the whole range for which platinum thermometers are useful, $-200°$ to $+1100°$, are given. From $-200°$ to $+200°$ the values are given for each degree,* and from $200°$ to $1100°$ for each $10°$.

For the figures given in Table I., in their final form, calculated by a method which renders unlikely the accumulation of errors greater than 1 unit in the last figure given, I am indebted to Mr. F. J. SELBY.

I have also to acknowledge my indebtedness to Mr. W. HUGO, who assisted during the comparisons by taking the observations with the platinum thermometers, and especially to the Director of the Laboratory, Dr. GLAZEBROOK, for his continued interest and advice on many points.

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* In the paper as printed this table has been shortened by only giving the value of $pt$ for intervals of $1°$ over the range $-50°$ to $+150°$.

## XXVI. Table I.

| pt. | T. | Difference for 1° pt. | Proportional parts. | pt. | T. | Difference for 1° pt. | Proportional parts. |
|---|---|---|---|---|---|---|---|
| − 200 | − 191·62 | | | − 39 | − 38·208 | | |
| | | ·934 | | | | ·974 | |
| − 190 | − 182·28 | | | − 38 | − 37·234 | | |
| | | ·936 | | | | ·975 | |
| − 180 | − 172·92 | | | − 37 | − 36·259 | | |
| | | ·939 | | | | ·975 | |
| − 170 | − 163·53 | | | − 36 | − 35·284 | | |
| | | ·941 | | | | ·975 | |
| − 160 | − 154·12 | | | − 35 | − 34·309 | | |
| | | ·943 | | | | ·975 | |
| − 150 | − 144·69 | | | − 34 | − 33·334 | | |
| | | ·946 | | | | ·976 | |
| − 140 | − 135·23 | | | − 33 | − 32·358 | | |
| | | ·949 | | | | ·976 | |
| − 130 | − 125·74 | | | − 32 | − 31·382 | | |
| | | ·951 | | | | ·976 | |
| − 120 | − 116·23 | | | − 31 | − 30·406 | | |
| | | ·954 | | | | ·977 | |
| − 110 | − 106·69 | | | − 30 | − 29·429 | | |
| | | ·956 | | | | ·977 | |
| − 100 | − 97·13 | | | − 29 | − 28·452 | | |
| | | ·959 | | | | ·977 | |
| − 90 | − 87·54 | | | − 28 | − 27·475 | | |
| | | ·962 | | | | ·978 | |
| − 80 | − 77·92 | | | − 27 | − 26·497 | | |
| | | ·964 | | | | ·978 | |
| − 70 | − 68·28 | | | − 26 | − 25·519 | | |
| | | ·967 | | | | ·978 | |
| − 60 | − 58·61 | | | − 25 | − 24·541 | | |
| | | ·970 | | | | ·978 | |
| − 50 | − 48·91 | | | − 24 | − 23·563 | | |
| | | ·971 | | | | ·979 | |
| − 49 | − 47·937 | | 970 | − 23 | − 22·584 | | |
| | | ·972 | | | | ·979 | |
| − 48 | − 46·965 | | | − 22 | − 21·605 | | |
| | | ·972 | ·1  ·097 | | | ·979 | |
| − 47 | − 45·993 | | | − 21 | − 20·626 | | |
| | | ·972 | ·2  ·194 | | | ·979 | |
| − 46 | − 45·021 | | | − 20 | − 19·647 | | |
| | | ·973 | ·3  ·291 | | | ·980 | |
| − 45 | − 44·048 | | | − 19 | − 18·667 | | 980 |
| | | ·973 | ·4  ·388 | | | ·980 | |
| − 44 | − 43·075 | | | − 18 | − 17·687 | | |
| | | ·973 | ·5  ·485 | | | ·980 | |
| − 43 | − 42·102 | | | − 17 | − 16·707 | | ·1  ·098 |
| | | ·973 | ·6  ·582 | | | ·980 | |
| − 42 | − 41·129 | | | − 16 | − 15·727 | | ·2  ·196 |
| | | ·973 | ·7  ·679 | | | ·981 | |
| − 41 | − 40·156 | | | − 15 | − 14·746 | | ·3  ·294 |
| | | ·974 | ·8  ·776 | | | ·981 | |
| − 40 | − 39·182 | | | − 14 | − 13·765 | | ·4  ·392 |
| | | ·974 | ·9  ·873 | | | ·981 | |
| − 39 | − 38·208 | | | − 13 | − 12·784 | | ·5  ·490 |
| | | ·974 | | | | ·982 | |

## TABLE I.—continued.

Left-hand section:

| pt. | T. | Difference for 1° pt. | Proportional parts. |
|---|---|---|---|
| − 13 | − 12·784 | | |
| | | ·982 | |
| − 12 | − 11·802 | | ·6   ·588 |
| | | ·982 | |
| − 11 | − 10·820 | | ·7   ·686 |
| | | ·982 | |
| − 10 | − 9·838 | | ·8   ·784 |
| | | ·983 | |
| − 9 | − 8·855 | | ·9   ·882 |
| | | ·983 | |
| − 8 | − 7·872 | | |
| | | ·983 | |
| − 7 | − 6·889 | | |
| | | ·983 | |
| − 6 | − 5·906 | | |
| | | ·984 | |
| − 5 | − 4·922 | | |
| | | ·984 | |
| − 4 | − 3·938 | | |
| | | ·984 | |
| − 3 | − 2·954 | | |
| | | ·984 | |
| − 2 | − 1·970 | | |
| | | ·985 | |
| − 1 | − 0·985 | | |
| | | ·985 | |
| + 0 | + 0·000 | | |
| | | ·985 | |
| + 1 | + 0·985 | | |
| | | ·986 | |
| + 2 | + 1·971 | | |
| | | ·986 | |
| + 3 | + 2·957 | | |
| | | ·986 | |
| + 4 | + 3·943 | | |
| | | ·987 | |
| + 5 | + 4·930 | | |
| | | ·987 | |
| + 6 | + 5·917 | | |
| | | ·987 | |
| + 7 | + 6·904 | | |
| | | ·987 | |
| + 8 | + 7·891 | | |
| | | ·988 | |
| + 9 | + 8·879 | | |
| | | ·988 | |
| + 10 | + 9·867 | | |
| | | ·988 | |
| + 11 | + 10·855 | | |
| | | ·988 | |
| + 12 | + 11·843 | | |
| | | ·988 | |
| + 13 | + 12·832 | | |
| | | ·989 | |

Right-hand section:

| pt. | T. | Difference for 1° pt. | Proportional parts. |
|---|---|---|---|
| + 13 | + 12·832 | | |
| | | ·989 | |
| + 14 | + 13·821 | | |
| | | ·989 | |
| + 15 | + 14·811 | | |
| | | ·989 | |
| + 16 | + 15·800 | | |
| | | ·990 | |
| + 17 | + 16·790 | | 990 |
| | | ·991 | |
| + 18 | + 17·781 | | |
| | | ·990 | |
| + 19 | + 18·771 | | ·1   ·099 |
| | | ·991 | |
| + 20 | + 19·762 | | ·2   ·198 |
| | | ·991 | |
| + 21 | + 20·753 | | ·3   ·297 |
| | | ·992 | |
| + 22 | + 21·745 | | ·4   ·396 |
| | | ·992 | |
| + 23 | + 22·737 | | ·5   ·495 |
| | | ·992 | |
| + 24 | + 23·729 | | ·6   ·594 |
| | | ·992 | |
| + 25 | + 24·721 | | ·7   ·693 |
| | | ·993 | |
| + 26 | + 25·714 | | ·8   ·792 |
| | | ·993 | |
| + 27 | + 26·707 | | ·9   ·891 |
| | | ·993 | |
| + 28 | + 27·700 | | |
| | | ·993 | |
| + 29 | + 28·693 | | |
| | | ·994 | |
| + 30 | + 29·687 | | |
| | | ·994 | |
| + 31 | + 30·681 | | |
| | | ·994 | |
| + 32 | + 31·675 | | |
| | | ·995 | |
| + 33 | + 32·670 | | |
| | | ·995 | |
| + 34 | + 33·665 | | |
| | | ·995 | |
| + 35 | + 34·660 | | |
| | | ·996 | |
| + 36 | + 35·656 | | |
| | | ·996 | |
| + 37 | + 36·652 | | |
| | | ·996 | |
| + 38 | + 37·648 | | |
| | | ·996 | |
| + 39 | + 38·644 | | |
| | | ·997 | |

## TABLE I.—continued.

| pt. | T. | Difference for 1° pt. | Proportional parts. | | pt. | T. | Difference for 1° pt. | Proportional parts. | |
|---|---|---|---|---|---|---|---|---|---|
| + 39 | + 38·664 | | | | + 65 | + 64·657 | | | |
| | | ·997 | | | | | 1·005 | | |
| + 40 | + 39·641 | | | | + 66 | + 65·662 | | | |
| | | ·997 | | | | | 1·005 | | |
| + 41 | + 40·638 | | | | + 67 | + 66·667 | | | |
| | | ·997 | | | | | 1·005 | | |
| + 42 | + 41·635 | | | | + 68 | + 67·672 | | | |
| | | ·998 | | | | | 1·005 | | |
| + 43 | + 42·633 | | | | + 69 | + 68·677 | | | |
| | | ·998 | | | | | 1·006 | | |
| + 44 | + 43·631 | | | | + 70 | + 69·683 | | | |
| | | ·998 | | | | | 1·006 | | |
| + 45 | + 44·629 | | | | + 71 | + 70·689 | | | |
| | | ·999 | | | | | 1·007 | | |
| + 46 | + 45·628 | | | | + 72 | + 71·696 | | | |
| | | ·999 | | | | | 1·007 | | |
| + 47 | + 46·627 | | 1000 | | + 73 | + 72·703 | | | |
| | | ·999 | | | | | 1·007 | | |
| + 48 | + 47·626 | | | | + 74 | + 73·710 | | | |
| | | ·999 | ·1 | ·100 | | | 1·007 | | |
| + 49 | + 48·625 | | | | + 75 | + 74·717 | | | |
| | | 1·000 | ·2 | ·200 | | | 1·008 | | |
| + 50 | + 49·625 | | | | + 76 | + 75·725 | | | |
| | | 1·000 | ·3 | ·300 | | | 1·008 | | |
| + 51 | + 50·625 | | | | + 77 | + 76·733 | | | |
| | | 1·000 | ·4 | ·400 | | | 1·008 | | |
| + 52 | + 51·625 | | | | + 78 | + 77·741 | | | |
| | | 1·001 | ·5 | ·500 | | | 1·008 | | |
| + 53 | + 52·625 | | | | + 79 | + 78·749 | | | |
| | | 1·001 | ·6 | ·600 | | | 1·009 | | |
| + 54 | + 53·627 | | | | + 80 | + 79·758 | | | |
| | | 1·001 | ·7 | ·700 | | | 1·009 | | |
| + 55 | + 54·628 | | | | + 81 | + 80·767 | | 1010 | |
| | | 1·002 | ·8 | ·800 | | | 1·009 | | |
| + 56 | + 55·630 | | | | + 82 | + 81·776 | | | |
| | | 1·002 | ·9 | ·900 | | | 1·010 | | |
| + 57 | + 56·632 | | | | + 83 | + 82·786 | | ·1 | ·101 |
| | | 1·002 | | | | | 1·010 | | |
| + 58 | + 57·634 | | | | + 84 | + 83·796 | | ·2 | ·202 |
| | | 1·002 | | | | | 1·011 | | |
| + 59 | + 58·636 | | | | + 85 | + 84·807 | | ·3 | ·303 |
| | | 1·003 | | | | | 1·010 | | |
| + 60 | + 59·639 | | | | + 86 | + 85·817 | | ·4 | ·404 |
| | | 1·003 | | | | | 1·011 | | |
| + 61 | + 60·642 | | | | + 87 | + 86·828 | | ·5 | ·505 |
| | | 1·003 | | | | | 1·011 | | |
| + 62 | + 61·645 | | | | + 88 | + 87·839 | | ·6 | ·606 |
| | | 1·004 | | | | | 1·012 | | |
| + 63 | + 62·649 | | | | + 89 | + 88·851 | | ·7 | ·707 |
| | | 1·004 | | | | | 1·012 | | |
| + 64 | + 63·653 | | | | + 90 | + 89·863 | | ·8 | ·808 |
| | | 1·004 | | | | | 1·012 | | |
| + 65 | + 64·657 | | | | + 91 | + 90·875 | | ·9 | ·909 |
| | | 1·005 | | | | | 1·013 | | |

## TABLE I.—continued.

| pt. | T. | Difference for 1° pt. | Proportional parts. | | pt. | T. | Difference for 1° pt. | Proportional parts. | |
|---|---|---|---|---|---|---|---|---|---|
| + 91 | + 90·875 | | | | +117 | +117·304 | | | |
| | | 1·013 | | | | | 1·021 | | |
| + 92 | + 91·888 | | | | +118 | +118·325 | | ·6 | ·612 |
| | | 1·013 | | | | | 1·021 | | |
| + 93 | + 92·901 | | | | +119 | +119·346 | | ·7 | ·714 |
| | | 1·013 | | | | | 1·022 | | |
| + 94 | + 93·914 | | | | +120 | +120·368 | | ·8 | ·816 |
| | | 1·014 | | | | | 1·021 | | |
| + 95 | + 94·928 | | | | +121 | +121·389 | | ·9 | ·918 |
| | | 1·014 | | | | | 1·022 | | |
| + 96 | + 95·942 | | | | +122 | +122·411 | | | |
| | | 1·014 | | | | | 1·023 | | |
| + 97 | + 96·956 | | | | +123 | +123·434 | | | |
| | | 1·014 | | | | | 1·023 | | |
| + 98 | + 97·970 | | | | +124 | +124·457 | | | |
| | | 1·015 | | | | | 1·023 | | |
| + 99 | + 98·985 | | | | +125 | +124·480 | | | |
| | | 1·015 | | | | | 1·023 | | |
| +100 | +100·000 | | | | +126 | +126·503 | | | |
| | | 1·015 | | | | | 1·023 | | |
| +101 | +101·015 | | | | +127 | +127·526 | | | |
| | | 1·016 | | | | | 1·024 | | |
| +102 | +102·031 | | | | +128 | +128·550 | | | |
| | | 1·016 | | | | | 1·025 | | |
| +103 | +103·047 | | | | +129 | +129·575 | | | |
| | | 1·016 | | | | | 1·024 | | |
| +104 | +104·063 | | | | +130 | +130·599 | | | |
| | | 1·017 | | | | | 1·025 | | |
| +105 | +105·080 | | | | +131 | +131·624 | | | |
| | | 1·017 | | | | | 1·026 | | |
| +106 | +106·097 | | | | +132 | +132·650 | | | |
| | | 1·017 | | | | | 1·025 | | |
| +107 | +107·114 | | | | +133 | +133·675 | | | |
| | | 1·018 | | | | | 1·026 | | |
| +108 | +108·132 | | | | +134 | +134·701 | | | |
| | | 1·018 | | | | | 1·026 | | |
| +109 | +109·150 | | | | +135 | +135·727 | | | |
| | | 1·018 | | | | | 1·027 | | |
| +110 | +110·168 | | | | +136 | +136·754 | | | |
| | | 1·018 | | | | | 1·027 | | |
| +111 | +111·186 | | 1020 | | +137 | +137·781 | | | |
| | | 1·019 | | | | | 1·027 | | |
| +112 | +112·205 | | | | +138 | +138·808 | | 1030 | |
| | | 1·019 | | | | | 1·028 | | |
| +113 | +113·224 | | ·1 | ·102 | +139 | +139·836 | | | |
| | | 1·020 | | | | | 1·027 | | |
| +114 | +114·244 | | ·2 | ·204 | +140 | +140·863 | | ·1 | ·103 |
| | | 1·020 | | | | | 1·028 | | |
| +115 | +115·264 | | ·3 | ·306 | +141 | +141·891 | | ·2 | ·206 |
| | | 1·020 | | | | | 1·029 | | |
| +116 | +116·284 | | ·4 | ·408 | +142 | +142·920 | | ·3 | ·309 |
| | | 1·020 | | | | | 1·029 | | |
| +117 | +117·304 | | ·5 | ·510 | +143 | +143·949 | | ·4 | ·412 |
| | | 1·021 | | | | | 1·029 | | |

TABLE I.—continued.

| pt. | T. | Difference for 1° pt. | Proportional parts. | | pt. | T. | Difference for 1° pt. | Proportional parts. |
|---|---|---|---|---|---|---|---|---|
| + 143 | + 143·949 | | | | + 340 | + 353·44 | | |
| | | 1·029 | | | | | 1·102 | |
| + 144 | + 144·978 | | ·5 | ·515 | + 350 | + 364·46 | | |
| | | 1·030 | | | | | 1·106 | |
| + 145 | + 146·008 | | ·6 | ·618 | + 360 | + 375·52 | | |
| | | 1·029 | | | | | 1·110 | |
| + 146 | + 147·037 | | ·7 | ·721 | + 370 | + 386·62 | | |
| | | 1·030 | | | | | 1·114 | |
| + 147 | + 148·067 | | ·8 | ·824 | + 380 | + 397·77 | | |
| | | 1·031 | | | | | 1·118 | |
| + 148 | + 149·098 | | ·9 | ·927 | + 390 | + 408·95 | | |
| | | 1·031 | | | | | 1·123 | |
| + 149 | + 150·129 | | | | + 400 | + 420·18 | | |
| | | 1·031 | | | | | 1·127 | |
| + 150 | + 151·16 | | | | + 410 | + 431·45 | | |
| | | 1·033 | | | | | 1·132 | |
| + 160 | + 161·49 | | | | + 420 | + 442·77 | | |
| | | 1·036 | | | | | 1·136 | |
| + 170 | + 171·85 | | | | + 430 | + 454·13 | | |
| | | 1·040 | | | | | 1·140 | |
| + 180 | + 182·25 | | | | + 440 | + 465·53 | | |
| | | 1·043 | | | | | 1·140 | |
| + 190 | + 192·68 | | | | + 450 | + 476·97 | | |
| | | 1·046 | | | | | 1·149 | |
| + 200 | + 203·14 | | | | + 460 | + 488·46 | | |
| | | 1·051 | | | | | 1·154 | |
| + 210 | + 213·65 | | | | + 470 | + 500·00 | | |
| | | 1·053 | | | | | 1·158 | |
| + 220 | + 224·18 | | | | + 480 | + 511·58 | | |
| | | 1·057 | | | | | 1·163 | |
| + 230 | + 234·75 | | | | + 490 | + 523·21 | | |
| | | 1·060 | | | | | 1·168 | |
| + 240 | + 245·35 | | | | + 500 | + 534·89 | | |
| | | 1·064 | | | | | 1·173 | |
| + 250 | + 255·99 | | | | + 510 | + 546·62 | | |
| | | 1·068 | | | | | 1·178 | |
| + 260 | + 266·67 | | | | + 520 | + 558·40 | | |
| | | 1·071 | | | | | 1·182 | |
| + 270 | + 277·38 | | | | + 530 | + 570·22 | | |
| | | 1·075 | | | | | 1·188 | |
| + 280 | + 288·13 | | | | + 540 | + 582·10 | | |
| | | 1·079 | | | | | 1·193 | |
| + 290 | + 298·92 | | | | + 550 | + 594·03 | | |
| | | 1·083 | | | | | 1·198 | |
| + 300 | + 309·75 | | | | + 560 | + 606·00 | | |
| | | 1·086 | | | | | 1·203 | |
| + 310 | + 320·61 | | | | + 570 | + 618·03 | | |
| | | 1·090 | | | | | 1·208 | |
| + 320 | + 331·51 | | | | + 580 | + 630·11 | | |
| | | 1·094 | | | | | 1·213 | |
| + 330 | + 342·46 | | | | + 590 | + 642·24 | | |
| | | 1·098 | | | | | 1·219 | |
| + 340 | + 353·44 | | | | + 600 | + 654·43 | | |
| | | 1·102 | | | | | 1·224 | |

## Table I.—continued.

| pt. | T. | Difference for 1° pt. | Proportional parts. | pt. | T. | Difference for 1° pt. | Proportional parts. |
|---|---|---|---|---|---|---|---|
| + 600 | + 654·43 |  |  | +  850 | +  979·11 |  |  |
|  |  | 1·224 |  |  |  | 1·390 |  |
| + 610 | + 666·67 |  |  | +  860 | +  993·01 |  |  |
|  |  | 1·230 |  |  |  | 1·399 |  |
| + 620 | + 678·97 |  |  | +  870 | + 1007·00 |  |  |
|  |  | 1·235 |  |  |  | 1·407 |  |
| + 630 | + 691·32 |  |  | +  880 | + 1021·07 |  |  |
|  |  | 1·241 |  |  |  | 1·416 |  |
| + 640 | + 703·73 |  |  | +  890 | + 1035·23 |  |  |
|  |  | 1·247 |  |  |  | 1·424 |  |
| + 650 | + 716·20 |  |  | +  900 | + 1049·47 |  |  |
|  |  | 1·253 |  |  |  | 1·433 |  |
| + 660 | + 728·73 |  |  | +  910 | + 1063·80 |  |  |
|  |  | 1·259 |  |  |  | 1·441 |  |
| + 670 | + 741·32 |  |  | +  920 | + 1078·21 |  |  |
|  |  | 1·264 |  |  |  | 1·450 |  |
| + 680 | + 753·96 |  |  | +  930 | + 1092·71 |  |  |
|  |  | 1·271 |  |  |  | 1·460 |  |
| + 690 | + 766·67 |  |  | +  940 | + 1107·31 |  |  |
|  |  | 1·277 |  |  |  | 1·469 |  |
| + 700 | + 779·44 |  |  | +  950 | + 1122·00 |  |  |
|  |  | 1·283 |  |  |  | 1·479 |  |
| + 710 | + 792·27 |  |  | +  960 | + 1136·79 |  |  |
|  |  | 1·290 |  |  |  | 1·490 |  |
| + 720 | + 805·17 |  |  | +  970 | + 1151·69 |  |  |
|  |  | 1·296 |  |  |  | 1·499 |  |
| + 730 | + 818·13 |  |  | +  980 | + 1166·68 |  |  |
|  |  | 1·303 |  |  |  | 1·508 |  |
| + 740 | + 831·16 |  |  | +  990 | + 1181·76 |  |  |
|  |  | 1·310 |  |  |  | 1·519 |  |
| + 750 | + 844·26 |  |  | + 1000 | + 1196·95 |  |  |
|  |  | 1·316 |  |  |  | 1·529 |  |
| + 760 | + 857·42 |  |  | + 1010 | + 1212·24 |  |  |
|  |  | 1·323 |  |  |  | 1·541 |  |
| + 770 | + 870·65 |  |  | + 1020 | + 1227·65 |  |  |
|  |  | 1·330 |  |  |  | 1·552 |  |
| + 780 | + 883·95 |  |  | + 1030 | + 1243·17 |  |  |
|  |  | 1·337 |  |  |  | 1·563 |  |
| + 790 | + 897·32 |  |  | + 1040 | + 1258·80 |  |  |
|  |  | 1·344 |  |  |  | 1·575 |  |
| + 800 | + 910·76 |  |  | + 1050 | + 1274·55 |  |  |
|  |  | 1·352 |  |  |  | 1·587 |  |
| + 810 | + 924·28 |  |  | + 1060 | + 1290·42 |  |  |
|  |  | 1·359 |  |  |  | 1·599 |  |
| + 820 | + 937·87 |  |  | + 1070 | + 1306·41 |  |  |
|  |  | 1·367 |  |  |  | 1·611 |  |
| + 830 | + 951·54 |  |  | + 1080 | + 1322·52 |  |  |
|  |  | 1·374 |  |  |  | 1·624 |  |
| + 840 | + 965·28 |  |  | + 1090 | + 1338·76 |  |  |
|  |  | 1·383 |  |  |  | 1·637 |  |
| + 850 | + 979·11 |  |  | + 1100 | + 1355·13 |  |  |
|  |  | 1·390 |  |  |  |  |  |

XXVII.  TABLE II.—*To Calculate the Change in* T *for a Given Small*
*Change in* δ.

| T. | Change in T for change of + ·01 in δ. | T. | Change in T for change of + ·01 in δ. |
|---|---|---|---|
| − 200 | + ·0600 | + 250 | + ·0375 |
| − 180 | + ·0504 | + 300 | + ·0600 |
| − 160 | + ·0416 | + 350 | + ·0875 |
| − 140 | + ·0336 | + 400 | + ·1200 |
| − 120 | + ·0264 | + 450 | + ·1575 |
| − 100 | + ·0200 | + 500 | + ·200 |
| − 80 | + ·0144 | + 550 | + ·247 |
| − 60 | + ·0096 | + 600 | + ·300 |
| − 40 | + ·0056 | + 650 | + ·357 |
| − 20 | + ·0024 | + 700 | + ·420 |
| − 0 | + ·0000 | + 750 | + ·487 |
| + 20 | − ·0016 | + 800 | + ·560 |
| + 40 | − ·0024 | + 850 | + ·637 |
| + 60 | − ·0024 | + 900 | + ·720 |
| + 80 | − ·0016 | + 950 | + ·807 |
| + 100 | − ·0000 | + 1000 | + ·900 |
| + 120 | + ·0024 | + 1050 | + ·997 |
| + 140 | + ·0056 | + 1100 | + 1·100 |
| + 160 | + ·0096 | + 1150 | + 1·207 |
| + 180 | + ·0144 | + 1200 | + 1·320 |
| + 200 | + ·0200 | + 1250 | + 1·437 |